PART I

CHAPTER P
Preparation for Calculus

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Section P.1 Graphs and Models

Solutions to Odd-Numbered Exercises

1. $y = -\frac{1}{2}x + 2$

x-intercept: $(4, 0)$

y-intercept: $(0, 2)$

Matches graph (b)

3. $y = 4 - x^2$

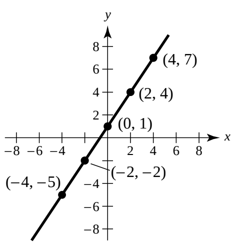
x-intercepts: $(2, 0), (-2, 0)$

y-intercept: $(0, 4)$

Matches graph (a)

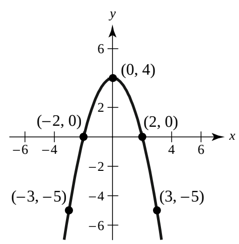
5. $y = \frac{3}{2}x + 1$

x	-4	-2	0	2	4
y	-5	-2	1	4	7



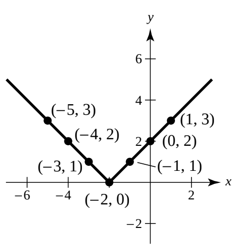
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



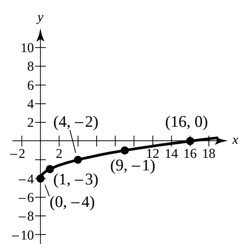
9. $y = |x + 2|$

x	-5	-4	-3	-2	-1	0	1
y	3	2	1	0	1	2	3



11. $y = \sqrt{x} - 4$

x	0	1	4	9	16
y	-4	-3	-2	-1	0

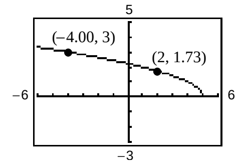


13.

Xmin = -3
Xmax = 5
Xscl = 1
Ymin = -3
Ymax = 5
Yscl = 1

Note that $y = 4$ when $x = 0$.

15.



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

17. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2;\ (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1;\ (-2, 0), (1, 0)$

19. $y = x^2\sqrt{25 - x^2}$

y-intercept: $y = 0^2\sqrt{25 - 0^2}$

$y = 0;\ (0, 0)$

x-intercepts: $0 = x^2\sqrt{25 - x^2}$

$0 = x^2\sqrt{(5 - x)(5 + x)}$

$x = 0, \pm 5;\ (0, 0); (\pm 5, 0)$

21. $y = \dfrac{3\left(2 - \sqrt{x}\right)}{x}$

y-intercept: None. x cannot equal 0.

x-intercepts: $0 = \dfrac{3\left(2 - \sqrt{x}\right)}{x}$

$0 = 2 - \sqrt{x}$

$x = 4;\ (4, 0)$

23. $x^2y - x^2 + 4y = 0$

y-intercept:

$0^2(y) - 0^2 + 4y = 0$

$y = 0;\ (0, 0)$

x-intercept:

$x^2(0) - x^2 + 4(0) = 0$

$x = 0;\ (0, 0)$

25. Symmetric with respect to the y-axis since

$y = (-x)^2 - 2 = x^2 - 2.$

27. Symmetric with respect to the x-axis since

$(-y)^2 = y^2 = x^3 - 4x.$

29. Symmetric with respect to the origin since

$(-x)(-y) = xy = 4.$

31. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

33. Symmetric with respect to the origin since

$$-y = \frac{-x}{(-x)^2 + 1}$$

$$y = \frac{x}{x^2 + 1}.$$

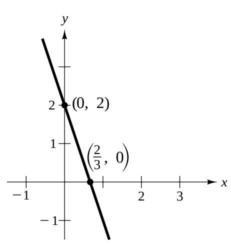
35. $y = |x^3 + x|$ is symmetric with respect to the y-axis since $y = |(-x)^3 + (-x)| = |-(x^3 + x)| = |x^3 + x|$.

37. $y = -3x + 2$

Intercepts:

$\left(\frac{2}{3}, 0\right), (0, 2)$

Symmetry: none

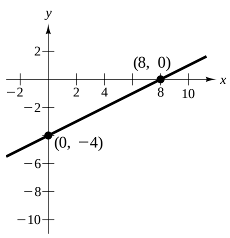


39. $y = \dfrac{x}{2} - 4$

Intercepts:

$(8, 0), (0, -4)$

Symmetry: none

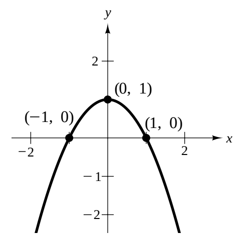


41. $y = 1 - x^2$

Intercepts:

$(1, 0), (-1, 0), (0, 1)$

Symmetry: y-axis

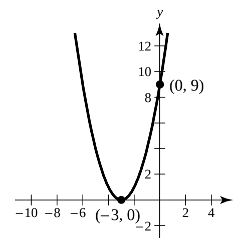


43. $y = (x + 3)^2$

Intercepts:

$(-3, 0), (0, 9)$

Symmetry: none

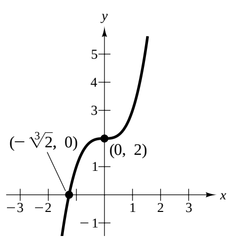


45. $y = x^3 + 2$

Intercepts:

$\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



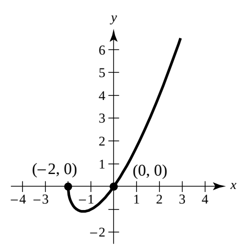
47. $y = x\sqrt{x + 2}$

Intercepts:

$(0, 0), (-2, 0)$

Symmetry: none

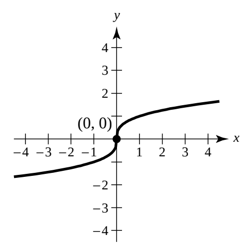
Domain: $x \geq -2$



49. $x = y^3$

Intercepts: $(0, 0)$

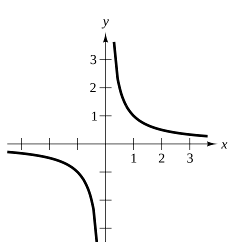
Symmetry: origin



51. $y = \dfrac{1}{x}$

Intercepts: none

Symmetry: origin

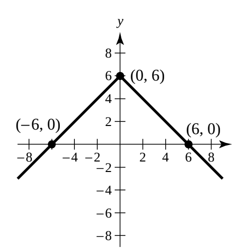


53. $y = 6 - |x|$

Intercepts:

$(0, 6), (-6, 0), (6, 0)$

Symmetry: y-axis



55. $y^2 - x = 9$

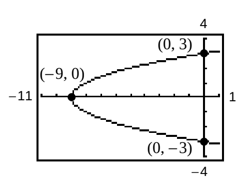
$y^2 = x + 9$

$y = \pm\sqrt{x + 9}$

Intercepts:

$(0, 3), (0, -3), (-9, 0)$

Symmetry: x-axis



57. $x + 3y^2 = 6$

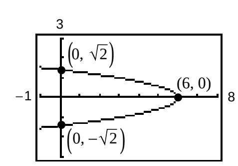
$3y^2 = 6 - x$

$y = \pm\sqrt{2 - \dfrac{x}{3}}$

Intercepts:

$(6, 0), \left(0, \sqrt{2}\right), \left(0, -\sqrt{2}\right)$

Symmetry: x-axis



59. $y = (x + 2)(x - 4)(x - 6)$ (other answers possible)

61. Some possible equations:

$y = x$

$y = x^3$

$y = 3x^3 - x$

$y = \sqrt[3]{x}$

63. $x + y = 2 \Rightarrow y = 2 - x$

$2x - y = 1 \Rightarrow y = 2x - 1$

$$\begin{aligned} 2 - x &= 2x - 1 \\ 3 &= 3x \\ 1 &= x \end{aligned}$$

The corresponding y-value is $y = 1$.

Point of intersection: $(1, 1)$

65. $x + y = 7 \Rightarrow y = 7 - x$

$3x - 2y = 11 \Rightarrow y = \dfrac{3x - 11}{2}$

$$\begin{aligned} 7 - x &= \frac{3x - 11}{2} \\ 14 - 2x &= 3x - 11 \\ -5x &= -25 \\ x &= 5 \end{aligned}$$

The corresponding y-value is $y = 2$.

Point of intersection: $(5, 2)$

67. $x^2 + y = 6 \Rightarrow y = 6 - x^2$

$x + y = 4 \Rightarrow y = 4 - x$

$$\begin{aligned} 6 - x^2 &= 4 - x \\ 0 &= x^2 - x - 2 \\ 0 &= (x - 2)(x + 1) \\ x &= 2, -1 \end{aligned}$$

The corresponding y-values are $y = 2$ (for $x = 2$) and $y = 5$ (for $x = -1$).

Points of intersection: $(2, 2), (-1, 5)$

69. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$$\begin{aligned} 5 - x^2 &= (x - 1)^2 \\ 5 - x^2 &= x^2 - 2x + 1 \\ 0 &= 2x^2 - 2x - 4 = 2(x + 1)(x - 2) \\ x &= -1 \text{ or } x = 2 \end{aligned}$$

The corresponding y-values are $y = -2$ and $y = 1$.

Points of intersection: $(-1, -2), (2, 1)$

71.
$$\begin{aligned} y &= x^3 \\ y &= x \\ x^3 &= x \\ x^3 - x &= 0 \\ x(x + 1)(x - 1) &= 0 \end{aligned}$$

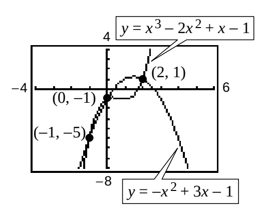
$x = 0, x = -1$, or $x = 1$

The corresponding y-values are $y = 0, y = -1$, and $y = 1$.

Points of intersection: $(0, 0), (-1, -1), (1, 1)$

73.
$$\begin{aligned} y &= x^3 - 2x^2 + x - 1 \\ y &= -x^2 + 3x - 1 \\ x^3 - 2x^2 + x - 1 &= -x^2 + 3x - 1 \\ x^3 - x^2 - 2x &= 0 \\ x(x - 2)(x + 1) &= 0 \\ x &= -1, 0, 2 \end{aligned}$$

$(-1, -5), (0, -1), (2, 1)$



75. $5.5\sqrt{x} + 10{,}000 = 3.29x$

$$\left(5.5\sqrt{x}\right)^2 = (3.29x - 10{,}000)^2$$

$$30.25x = 10.8241x^2 - 65{,}800x + 100{,}000{,}000$$

$$0 = 10.8241x^2 - 65{,}830.25x + 100{,}000{,}000 \qquad \text{Use the Quadratic Formula.}$$

$$x \approx 3133 \text{ units}$$

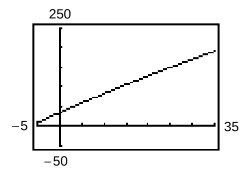
The other root, $x \approx 2949$, does not satisfy the equation $R = C$.

This problem can also be solved by using a graphing utility and finding the intersection of the graphs of C and R.

77. (a) Using a graphing utility, you obtain

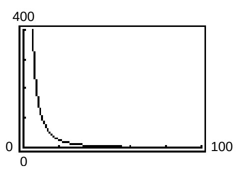
$$y = -0.0153t^2 + 4.9971t + 34.9405$$

(b)



(c) For the year 2004, $t = 34$ and $y \approx 187.2$ CPI.

79.



If the diameter is doubled, the resistance is changed by approximately a factor of $(1/4)$. For instance, $y(20) \approx 26.555$ and $y(40) \approx 6.36125$.

81. False; x-axis symmetry means that if $(1, -2)$ is on the graph, then $(1, 2)$ is also on the graph.

83. True; the x-intercepts are

$$\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right).$$

85. Distance to the origin $= K \times$ Distance to $(2, 0)$

$$\sqrt{x^2 + y^2} = K\sqrt{(x - 2)^2 + y^2},\ K \neq 1$$

$$x^2 + y^2 = K^2(x^2 - 4x + 4 + y^2)$$

$$(1 - K^2)x^2 + (1 - K^2)y^2 + 4K^2x - 4K^2 = 0$$

Note: This is the equation of a circle!

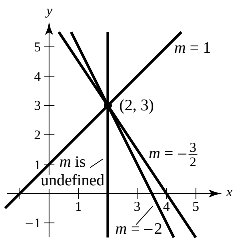
Section P.2 Linear Models and Rates of Change

1. $m = 1$

3. $m = 0$

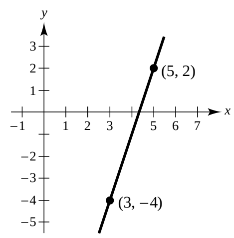
5. $m = -12$

7.



9. $m = \dfrac{2 - (-4)}{5 - 3}$

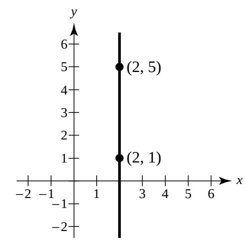
$= \dfrac{6}{2} = 3$



11. $m = \dfrac{5 - 1}{2 - 2}$

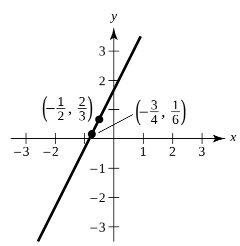
$= \dfrac{4}{0}$

undefined



13. $m = \dfrac{2/3 - 1/6}{-1/2 - (-3/4)}$

$= \dfrac{1/2}{1/4} = 2$



15. Since the slope is 0, the line is horizontal and its equation is $y = 1$. Therefore, three additional points are $(0, 1)$, $(1, 1)$, and $(3, 1)$.

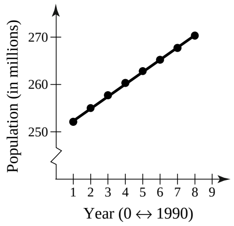
17. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

Therefore, three additional points are $(0, 10)$, $(2, 4)$, and $(3, 1)$.

19. Given a line L, you can use any two distinct points to calculate its slope. Since a line is straight, the ratio of the change in y-values to the change in x-values will always be the same. See Section P.2 Exercise 93 for a proof.

21. (a)



(b) The slopes of the line segments are

$$\frac{255.0 - 252.1}{2 - 1} = 2.9$$

$$\frac{257.7 - 255.0}{3 - 2} = 2.7$$

$$\frac{260.3 - 257.7}{4 - 3} = 2.6$$

$$\frac{262.8 - 260.3}{5 - 4} = 2.5$$

$$\frac{265.2 - 262.8}{6 - 5} = 2.4$$

$$\frac{267.7 - 265.2}{7 - 6} = 2.5$$

$$\frac{270.3 - 267.7}{8 - 7} = 2.6$$

The population increased most rapidly from 1991 to 1992.

$(m = 2.9)$

23. $x + 5y = 20$

$$y = -\tfrac{1}{5}x + 4$$

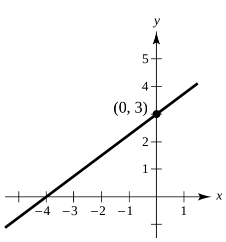
Therefore, the slope is $m = -\frac{1}{5}$ and the y-intercept is $(0, 4)$.

25. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

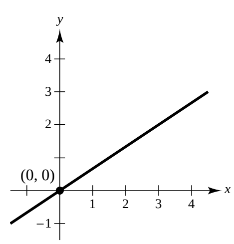
27.

$$y = \tfrac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$



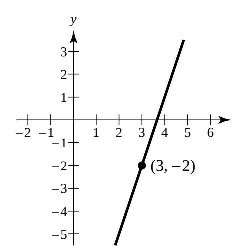
29.

$$y = \tfrac{2}{3}x$$
$$3y = 2x$$
$$2x - 3y = 0$$



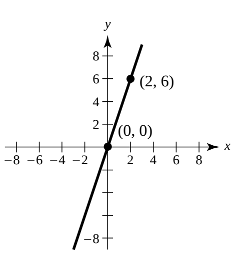
31.

$$y + 2 = 3(x - 3)$$
$$y + 2 = 3x - 9$$
$$y = 3x - 11$$
$$y - 3x + 11 = 0$$



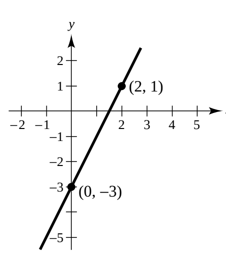
33. $m = \dfrac{6 - 0}{2 - 0} = 3$

$$y - 0 = 3(x - 0)$$
$$y = 3x$$



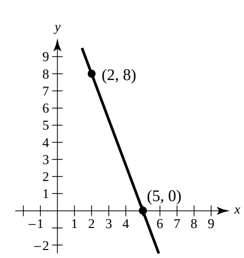
35. $m = \dfrac{1 - (-3)}{2 - 0} = 2$

$$y - 1 = 2(x - 2)$$
$$y - 1 = 2x - 4$$
$$0 = 2x - y - 3$$



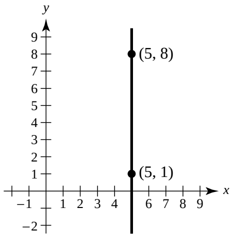
37. $m = \dfrac{8 - 0}{2 - 5} = -\dfrac{8}{3}$

$$y - 0 = -\frac{8}{3}(x - 5)$$
$$y = -\frac{8}{3}x + \frac{40}{3}$$
$$3y + 8x - 40 = 0$$



39. $m = \dfrac{8 - 1}{5 - 5}$ Undefined.

Vertical line $x = 5$

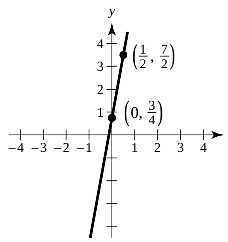


41. $m = \dfrac{7/2 - 3/4}{1/2 - 0} = \dfrac{11/4}{1/2} = \dfrac{11}{2}$

$$y - \frac{3}{4} = \frac{11}{2}(x - 0)$$

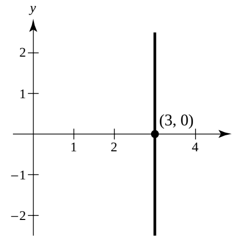
$$y = \frac{11}{2}x + \frac{3}{4}$$

$22x - 4y + 3 = 0$



43. $x = 3$

$x - 3 = 0$



45. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

47. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$$\frac{1}{a} + \frac{2}{a} = 1$$

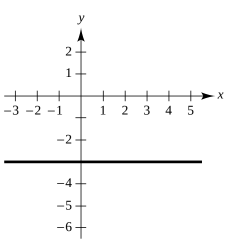
$$\frac{3}{a} = 1$$

$$a = 3 \Longrightarrow x + y = 3$$

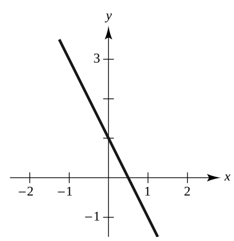
$$x + y - 3 = 0$$

49. $y = -3$

$y + 3 = 0$



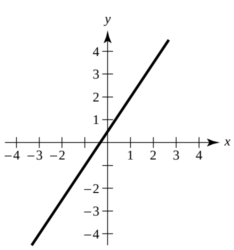
51. $y = -2x + 1$



53. $y - 2 = \frac{3}{2}(x - 1)$

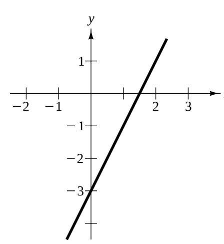
$$y = \tfrac{3}{2}x + \tfrac{1}{2}$$

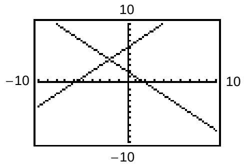
$2y - 3x - 1 = 0$



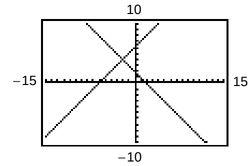
55. $2x - y - 3 = 0$

$$y = 2x - 3$$



57. 

The lines do not appear perpendicular.



The lines appear perpendicular.

The lines are perpendicular because their slopes 1 and -1 are negative reciprocals of each other. You must use a square setting in order for perpendicular lines to appear perpendicular.

59. $4x - 2y = 3$

$$y = 2x - \tfrac{3}{2}$$

$$m = 2$$

(a) $$y - 1 = 2(x - 2)$$
$$y - 1 = 2x - 4$$
$$2x - y - 3 = 0$$

(b) $$y - 1 = -\tfrac{1}{2}(x - 2)$$
$$2y - 2 = -x + 2$$
$$x + 2y - 4 = 0$$

61. $5x - 3y = 0$

$$y = \tfrac{5}{3}x$$

$$m = \tfrac{5}{3}$$

(a) $$y - \tfrac{7}{8} = \tfrac{5}{3}\left(x - \tfrac{3}{4}\right)$$
$$24y - 21 = 40x - 30$$
$$24y - 40x + 9 = 0$$

(b) $$y - \tfrac{7}{8} = -\tfrac{3}{5}\left(x - \tfrac{3}{4}\right)$$
$$40y - 35 = -24x + 18$$
$$40y + 24x - 53 = 0$$

63. (a) $x = 2 \Longrightarrow x - 2 = 0$

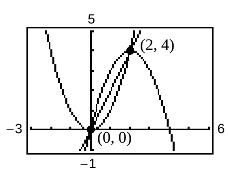
(b) $y = 5 \Longrightarrow y - 5 = 0$

65. The slope is 125. Hence, $V = 125(t - 1) + 2540$

$$= 125t + 2415$$

67. The slope is -2000. Hence, $V = -2000(t - 1) + 20{,}400$

$$= -2000t + 22{,}400$$

69. 

You can use the graphing utility to determine that the points of intersection are $(0, 0)$ and $(2, 4)$. Analytically,

$$x^2 = 4x - x^2$$
$$2x^2 - 4x = 0$$
$$2x(x - 2) = 0$$
$$x = 0 \Longrightarrow y = 0 \Longrightarrow (0, 0)$$
$$x = 2 \Longrightarrow y = 4 \Longrightarrow (2, 4).$$

The slope of the line joining $(0, 0)$ and $(2, 4)$ is $m = (4 - 0)/(2 - 0) = 2$. Hence, an equation of the line is

$$y - 0 = 2(x - 0)$$
$$y = 2x.$$

71. $m_1 = \dfrac{1 - 0}{-2 - (-1)} = -1$

$m_2 = \dfrac{-2 - 0}{2 - (-1)} = -\dfrac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

73. Equations of perpendicular bisectors:

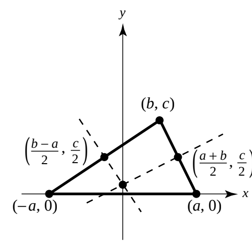
$$y - \frac{c}{2} = \frac{a - b}{c}\left(x - \frac{a + b}{2}\right)$$

$$y - \frac{c}{2} = \frac{a + b}{-c}\left(x - \frac{b - a}{2}\right)$$

Letting $x = 0$ in either equation gives the point of intersection:

$$\left(0, \frac{-a^2 + b^2 + c^2}{2c}\right).$$

This point lies on the third perpendicular bisector, $x = 0$.



75. Equations of altitudes:

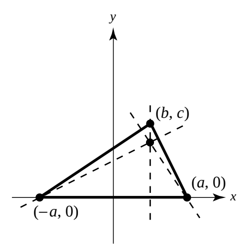
$$y = \frac{a - b}{c}(x + a)$$

$$x = b$$

$$y = -\frac{a + b}{c}(x - a)$$

Solving simultaneously, the point of intersection is

$$\left(b, \frac{a^2 - b^2}{c}\right).$$



77. Find the equation of the line through the points $(0, 32)$ and $(100, 212)$.

$$m = \tfrac{180}{100} = \tfrac{9}{5}$$

$$F - 32 = \tfrac{9}{5}(C - 0)$$

$$F = \tfrac{9}{5}C + 32$$

$$5F - 9C - 160 = 0$$

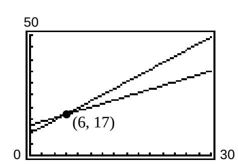
For $F = 72°$, $C \approx 22.2°$.

79. (a) $W_1 = 0.75x + 12.50$

$W_2 = 1.30x + 9.20$

(c) Both jobs pay \$17 per hour if 6 units are produced. For someone who can produce more than 6 units per hour, the second offer would pay more. For a worker who produces less than 6 units per hour, the first offer pays more.

(b)



Using a graphing utility, the point of intersection is approximately $(6, 17)$. Analytically,

$$0.75x + 12.50 = 1.30x + 9.20$$

$$3.3 = 0.55x \Longrightarrow x = 6$$

$$y = 0.75(6) + 12.50 = 17.$$

81. (a) Two points are $(50, 580)$ and $(47, 625)$. The slope is

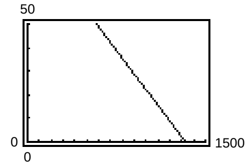
$$m = \frac{625 - 580}{47 - 50} = -15.$$

$$p - 580 = -15(x - 50)$$

$$p = -15x + 750 + 580 = -15x + 1330$$

$$\text{or } x = \tfrac{1}{15}(1330 - p)$$

(b)



If $p = 655$, $x = \frac{1}{15}(1330 - 655) = 45$ units.

(c) If $p = 595$, $x = \frac{1}{15}(1330 - 595) = 49$ units.

83. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{|4(0) + 3(0) - 10|}{\sqrt{4^2 + 3^2}} = \dfrac{10}{5} = 2$

85. $x - y - 2 = 0 \Rightarrow d = \dfrac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5\sqrt{2}}{2}$

87. A point on the line $x + y = 1$ is $(0, 1)$. The distance from the point $(0, 1)$ to $x + y - 5 = 0$ is

$$d = \frac{|1(0) + 1(1) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|1 - 5|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

89. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.)

The slope of the line $Ax + By + C = 0$ is $-A/B$. The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$y - y_1 = \frac{B}{A}(x - x_1)$$

$$Ay - Ay_1 = Bx - Bx_1$$

$$Bx_1 - Ay_1 = Bx - Ay$$

The point of intersection of these two lines is:

$$Ax + By = -C \quad\Rightarrow\quad A^2x + ABy = -AC \qquad (1)$$

$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} \quad (2)$$

$$(A^2 + B^2)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))}$$

$$x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

$$Ax + By = -C \quad\Rightarrow\quad ABx + B^2y = -BC \qquad (3)$$

$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{-AB_x + A^2y} = \underline{-ABx_1 + A^2y_1} \quad (4)$$

$$(A^2 + B^2)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))}$$

$$y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}$$

—CONTINUED—

89. —CONTINUED—

$$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right) \text{ point of intersection}$$

The distance between (x_1, y_1) and this point gives us the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$\begin{aligned} d &= \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2} \\ &= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2} \\ &= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} \\ &= \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}} \\ &= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \end{aligned}$$

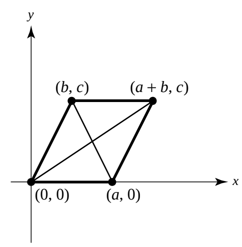
91. For simplicity, let the vertices of the rhombus be $(0, 0)$, $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure. The slopes of the diagonals are then

$$m_1 = \frac{c}{a + b} \text{ and } m_2 = \frac{c}{b - a}.$$

Since the sides of the Rhombus are equal, $a^2 = b^2 + c^2$, and we have

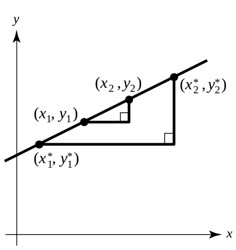
$$m_1m_2 = \frac{c}{a + b} \cdot \frac{c}{b - a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



93. Consider the figure below in which the four points are collinear. Since the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



95. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \quad \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

Section P.3 Functions and Their Graphs

1. (a) $f(0) = 2(0) - 3 = -3$

(b) $f(-3) = 2(-3) - 3 = -9$

(c) $f(b) = 2b - 3$

(d) $f(x - 1) = 2(x - 1) - 3 = 2x - 5$

3. (a) $g(0) = 3 - 0^2 = 3$

(b) $g(\sqrt{3}) = 3 - (\sqrt{3})^2 = 3 - 3 = 0$

(c) $g(-2) = 3 - (-2)^2 = 3 - 4 = -1$

(d) $g(t - 1) = 3 - (t - 1)^2 = -t^2 + 2t + 2$

5. (a) $f(0) = \cos(2(0)) = \cos 0 = 1$

(b) $f\left(-\frac{\pi}{4}\right) = \cos\left(2\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

(c) $f\left(\frac{\pi}{3}\right) = \cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

7. $$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2, \Delta x \neq 0$$

9. $$\frac{f(x) - f(2)}{x - 2} = \frac{(1/\sqrt{x - 1} - 1)}{x - 2}$$

$$= \frac{1 - \sqrt{x - 1}}{(x - 2)\sqrt{x - 1}} \cdot \frac{1 + \sqrt{x - 1}}{1 + \sqrt{x - 1}} = \frac{2 - x}{(x - 2)\sqrt{x - 1}(1 + \sqrt{x - 1})} = \frac{-1}{\sqrt{x - 1}(1 + \sqrt{x - 1})}, x \neq 2$$

11. $h(x) = -\sqrt{x + 3}$

Domain: $x + 3 \geq 0 \Longrightarrow [-3, \infty)$

Range: $(-\infty, 0]$

13. $f(t) = \sec\frac{\pi t}{4}$

$\frac{\pi t}{4} \neq \frac{(2k + 1)\pi}{2} \Longrightarrow t \neq 4k + 2$

Domain: all $t \neq 4k + 2$, k an integer

Range: $(-\infty, -1], [1, \infty)$

15. $f(x) = \frac{1}{x}$

Domain: $(-\infty, 0), (0, \infty)$

Range: $(-\infty, 0), (0, \infty)$

17. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) = 2t^2 + 4$

(Note: $t^2 + 1 \geq 0$ for all t)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1), [2, \infty)$

19. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

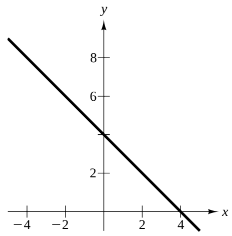
Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

21. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

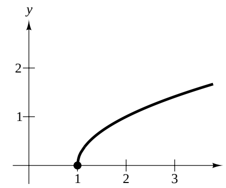
Range: $(-\infty, \infty)$



23. $h(x) = \sqrt{x - 1}$

Domain: $[1, \infty)$

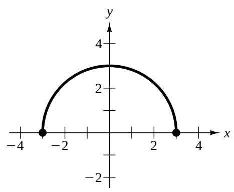
Range: $[0, \infty)$



25. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

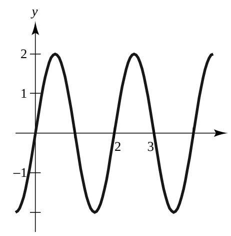
Range: $[0, 3]$



27. $g(t) = 2 \sin \pi t$

Domain: $(-\infty, \infty)$

Range: $[-2, 2]$



29. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

31. y is a function of x. Vertical lines intersect the graph at most once.

33. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

y is not a function of x since there are two values of y for some x.

35. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x since there are two values of y for some x.

37. $f(x) = |x| + |x - 2|$

If $x < 0$, then $f(x) = -x - (x - 2) = -2x + 2 = 2(1 - x)$.

If $0 \le x < 2$, then $f(x) = x - (x - 2) = 2$.

If $x \ge 2$, then $f(x) = x + (x - 2) = 2x - 2 = 2(x - 1)$.

Thus,

$$f(x) = \begin{cases} 2(1 - x), & x < 0 \\ 2, & 0 \le x < 2. \\ 2(x - 1), & x \ge 2. \end{cases}$$

39. The function is $g(x) = cx^2$. Since $(1, -2)$ satisfies the equation, $c = -2$. Thus, $g(x) = -2x^2$.

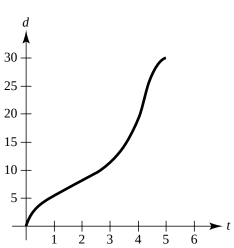
41. The function is $r(x) = c/x$, since it must be undefined at $x = 0$. Since $(1, 32)$ satisfies the equation, $c = 32$. Thus, $r(x) = 32/x$.

43. (a) For each time t, there corresponds a depth d.

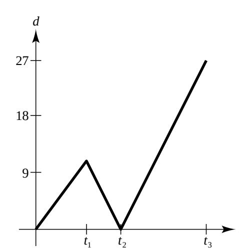
(b) Domain: $0 \le t \le 5$

Range: $0 \le d \le 30$

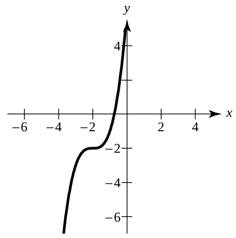
(c)



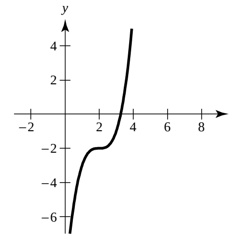
45.



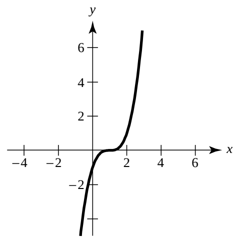
47. (a) The graph is shifted 3 units to the left.



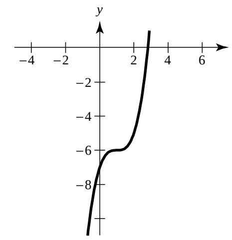
(b) The graph is shifted 1 unit to the right.



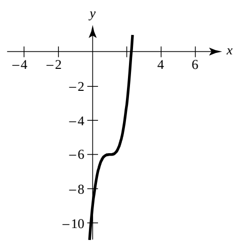
(c) The graph is shifted 2 units upward.



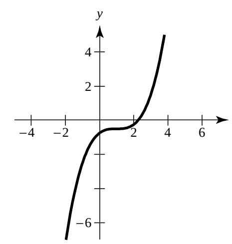
(d) The graph is shifted 4 units downward.



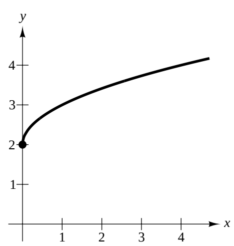
(e) The graph is stretched vertically by a factor of 3.



(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.

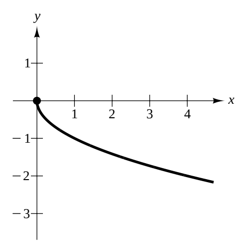


49. (a) $y = \sqrt{x} + 2$



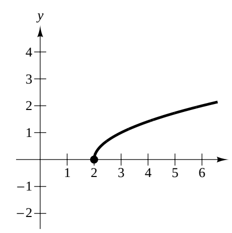
Vertical shift 2 units upward

(b) $y = -\sqrt{x}$



Reflection about the x-axis

(c) $y = \sqrt{x - 2}$



Horizontal shift 2 units to the right

51. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the program would turn on (and off) one hour later.

(c) If $H(t) = T(t) - 1$, then the overall temperature would be reduced 1 degree.

53. $f(x) = x^2, g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x, \quad x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

55. $f(x) = \dfrac{3}{x}, g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \dfrac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{3}{x}\right) = \left(\dfrac{3}{x}\right)^2 - 1 = \dfrac{9}{x^2} - 1 = \dfrac{9 - x^2}{x^2}$

Domain: all $x \neq 0$

No, $f \circ g \neq g \circ f$.

57. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ represents the area of the circle at time t.

59. $f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$

Even

61. $f(-x) = (-x)\cos(-x) = -x\cos x = -f(x)$

Odd

63. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

65.
$$\begin{aligned} f(-x) &= a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x) \\ &= -[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x] \\ &= -f(x) \end{aligned}$$

Odd

67. Let $F(x) = f(x)g(x)$ where f and g are even. Then

$$F(-x) = f(-x)g(-x) = f(x)g(x) = F(x).$$

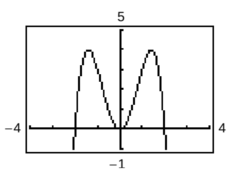
Thus, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

$$F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x).$$

Thus, $F(x)$ is even.

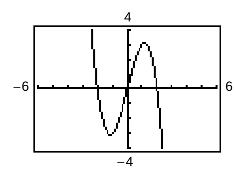
69. $f(x) = x^2 + 1$ and $g(x) = x^4$ are even.

$f(x)g(x) = (x^2 + 1)(x^4) = x^6 + x^4$ is even.



$f(x) = x^3 - x$ is odd and $g(x) = x^2$ is even.

$f(x)g(x) = (x^3 - x)(x^2) = x^5 - x^3$ is odd.

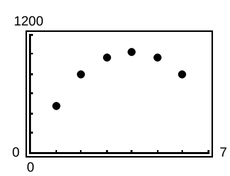


71. (a)

x	*length and width*	*volume V*
1	$24 - 2(1)$	484
2	$24 - 2(2)$	800
3	$24 - 2(3)$	972
4	$24 - 2(4)$	1024
5	$24 - 2(5)$	980
6	$24 - 2(6)$	864

The maximum volume appears to be 1024 cm^3.

(b)

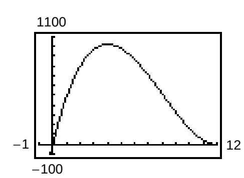


Yes, V is a function of x.

(c) $V = x(24 - 2x)^2 = 4x(12 - x)^2$

Domain: $0 < x < 12$

(d)



Maximum volume is $V = 1024\text{ cm}^3$ for box having dimensions $4 \times 16 \times 16$ cm.

73. False; let $f(x) = x^2$.

Then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

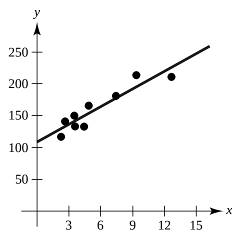
75. True, the function is even.

Section P.4 Fitting Models to Data

1. Quadratic function

3. Linear function

5. (a), (b)

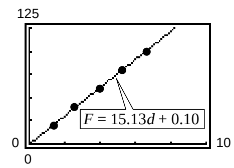


Yes. The cancer mortality increases linearly with increased exposure to the carcinogenic substance.

(c) If $x = 3$, then $y \approx 136$.

7. (a) $d = 0.066F$ or $F = 15.1d + 0.1$

(b)



The model fits well.

(c) If $F = 55$, then $d \approx 0.066(55) = 3.63$ cm.

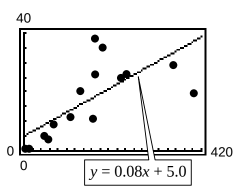
9. (a) Let x = per capita energy usage (in millions of Btu)

y = per capita gross national product (in thousands)

$y = 0.0764x + 4.9985 \approx 0.08x + 5.0$

$r = 0.7052$

(b)



(c) Denmark, Japan, and Canada

(d) Deleting the data for the three countries above,

$y = 0.0959x + 1.0539$

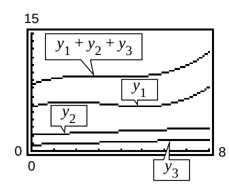
($r = 0.9202$ is much closer to 1.)

11. (a) $y_1 = 0.0343t^3 - 0.3451t^2 + 0.8837t + 5.6061$

$y_2 = 0.1095t + 2.0667$

$y_3 = 0.0917t + 0.7917$

(b)

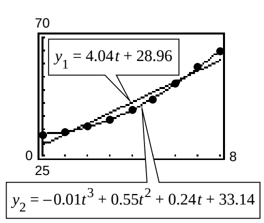


For 2002, $t = 12$ and $y_1 + y_2 + y_3 \approx 31.06$ cents/mile

13. (a) $y_1 = 4.0367t + 28.9644$

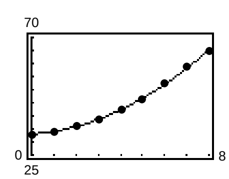
$y_2 = -0.0099t^3 + 0.5488t^2 + 0.2399t + 33.1414$

(b)



(c) The cubic model is better.

(d) $y_3 = 0.4297t^2 + 0.5994t + 32.9745$



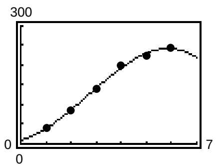
(e) The slope represents the average increase per year in the number of people (in millions) in HMOs.

(f) For 2000, $t = 10$, and $y_1 \approx 69.3$ million. (linear)

$y_2 \approx 80.5$ million (cubic)

15. (a) $y = -1.81x^3 + 14.58x^2 + 16.39x + 10$

(b)



(c) If $x = 4.5$, $y \approx 214$ horsepower.

17. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately

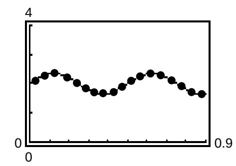
$$(2.35 - 1.65)/2 = 0.35.$$

The period is approximately

$$2(0.375 - 0.125) = 0.5.$$

(c) One model is $y = 0.35\sin(4\pi t) + 2$.

(d)



19. Answers will vary.

Review Exercises for Chapter P

1. $y = 2x - 3$

$x = 0 \Rightarrow y = 2(0) - 3 = -3 \Rightarrow (0, -3)$ y-intercept

$y = 0 \Rightarrow 0 = 2x - 3 \Rightarrow x = \frac{3}{2} \Rightarrow \left(\frac{3}{2}, 0\right)$ x-intercept

3. $y = \dfrac{x-1}{x-2}$

$x = 0 \Rightarrow y = \dfrac{0-1}{0-2} = \dfrac{1}{2} \Rightarrow \left(0, \dfrac{1}{2}\right)$ y-intercept

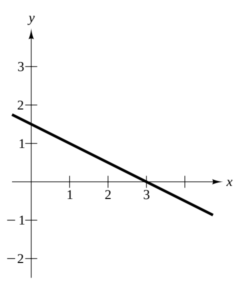
$y = 0 \Rightarrow 0 = \dfrac{x-1}{x-2} \Rightarrow x = 1 \Rightarrow (1, 0)$ x-intercept

5. Symmetric with respect to y-axis since

$$(-x)^2y - (-x)^2 + 4y = 0$$

$$x^2y - x^2 + 4y = 0.$$

7. $y = -\frac{1}{2}x + \frac{3}{2}$



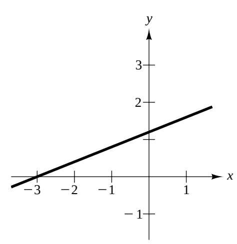
9. $-\frac{1}{3}x + \frac{5}{6}y = 1$

$-\frac{2}{5}x + y = \frac{6}{5}$

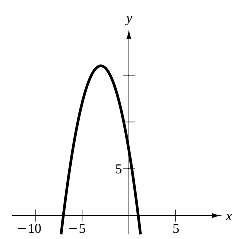
$y = \frac{2}{5}x + \frac{6}{5}$

Slope: $\frac{2}{5}$

y-intercept: $\frac{6}{5}$

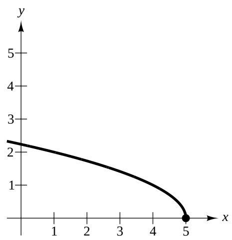


11. $y = 7 - 6x - x^2$



13. $y = \sqrt{5 - x}$

Domain: $(-\infty, 5]$



15. $y = 4x^2 - 25$

Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -30
Ymax = 10
Yscl = 5

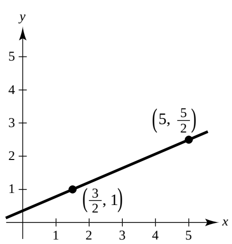
17.
$$\begin{aligned} 3x - 4y &= 8 \\ 4x + 4y &= 20 \\ \hline 7x \quad &= 28 \\ x &= 4 \\ y &= 1 \end{aligned}$$

Point: $(4, 1)$

19. You need factors $(x + 2)$ and $(x - 2)$. Multiply by x to obtain origin symmetry

$$\begin{aligned} y &= x(x + 2)(x - 2). \\ &= x^3 - 4x. \end{aligned}$$

21.

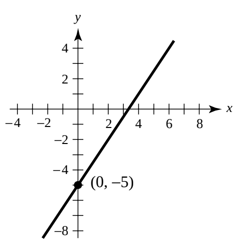


Slope $= \dfrac{(5/2) - 1}{5 - (3/2)} = \dfrac{3/2}{7/2} = \dfrac{3}{7}$

23.
$$\begin{aligned} \frac{1 - t}{1 - 0} &= \frac{1 - 5}{1 - (-2)} \\ 1 - t &= -\frac{4}{3} \\ t &= \frac{7}{3} \end{aligned}$$

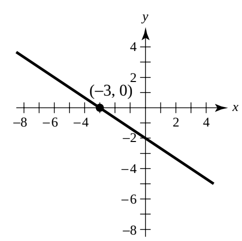
25.
$$\begin{aligned} y - (-5) &= \tfrac{3}{2}(x - 0) \\ y &= \tfrac{3}{2}x - 5 \end{aligned}$$

$2y - 3x + 10 = 0$



27.
$$\begin{aligned} y - 0 &= -\tfrac{2}{3}(x - (-3)) \\ y &= -\tfrac{2}{3}x - 2 \end{aligned}$$

$3y + 2x + 6 = 0$



29. (a) $y - 4 = \frac{7}{16}(x + 2)$

$16y - 64 = 7x + 14$

$0 = 7x - 16y + 78$

(b) Slope of line is $\frac{5}{3}$.

$y - 4 = \frac{5}{3}(x + 2)$

$3y - 12 = 5x + 10$

$0 = 5x - 3y + 22$

(c) $m = \frac{4 - 0}{-2 - 0} = -2$

$y = -2x$

$2x + y = 0$

(d) $x = -2$

$x + 2 = 0$

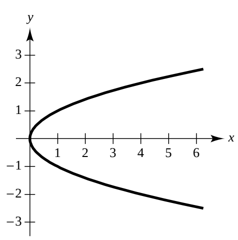
31. The slope is -850. $V = -850t + 12{,}500$.

$V(3) = -850(3) + 12{,}500 = \$9950$

33. $x - y^2 = 0$

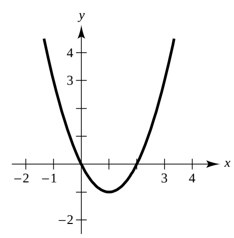
$y = \pm\sqrt{x}$

Not a function of x since there are two values of y for some x.



35. $y = x^2 - 2x$

Function of x since there is one value of y for each x.



37. $f(x) = \frac{1}{x}$

(a) $f(0)$ does not exist.

(b) $\frac{f(1 + \Delta x) - f(1)}{\Delta x} = \frac{\frac{1}{1 + \Delta x} - \frac{1}{1}}{\Delta x} = \frac{1 - 1 - \Delta x}{(1 + \Delta x)\Delta x}$

$= \frac{-1}{1 + \Delta x}, \Delta x \neq -1, 0$

39. (a) Domain: $36 - x^2 \geq 0 \Rightarrow -6 \leq x \leq 6$ or $[-6, 6]$

Range: $[0, 6]$

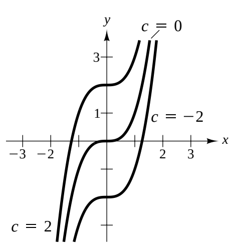
(b) Domain: all $x \neq 5$ or $(-\infty, 5), (5, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0), (0, \infty)$

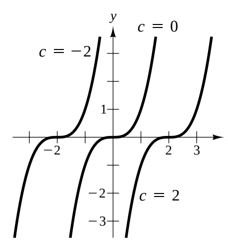
(c) Domain: all x or $(-\infty, \infty)$

Range: all y or $(-\infty, \infty)$

41. (a) $f(x) = x^3 + c$, $c = -2, 0, 2$



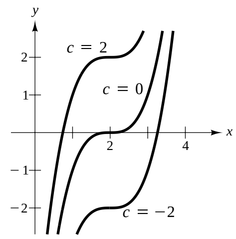
(b) $f(x) = (x - c)^3$, $c = -2, 0, 2$



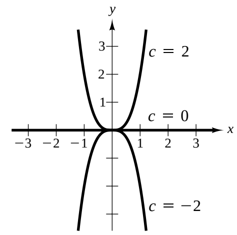
—CONTINUED—

41. —CONTINUED—

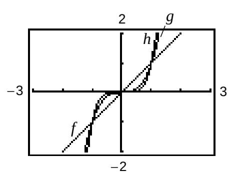
(c) $f(x) = (x - 2)^3 + c$, $c = -2, 0, 2$



(d) $f(x) = cx^3$, $c = -2, 0, 2$

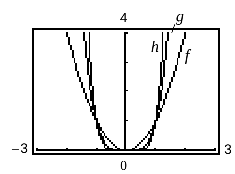


43. (a) Odd powers: $f(x) = x$, $g(x) = x^3$, $h(x) = x^5$



The graphs of f, g, and h all rise to the right and fall to the left. As the degree increases, the graph rises and falls more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, -1)$.

Even powers: $f(x) = x^2$, $g(x) = x^4$, $h(x) = x^6$

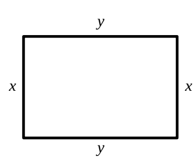


The graphs of f, g, and h all rise to the left and to the right. As the degree increases, the graph rises more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, 1)$.

(b) $y = x^7$ will look like $h(x) = x^5$, but rise and fall even more steeply.

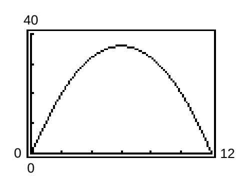
$y = x^8$ will look like $h(x) = x^6$, but rise even more steeply.

45. (a)



$$2x + 2y = 24$$
$$y = 12 - x$$
$$A = xy = x(12 - x) = 12x - x^2$$

(b) Domain: $0 < x < 12$



(c) Maximum area is $A = 36$. In general, the maximum area is attained when the rectangle is a square. In this case, $x = 6$.

47. (a) 3 (cubic), negative leading coefficient

(b) 4 (quartic), positive leading coefficient

(c) 2 (quadratic), negative leading coefficient

(d) 5, positive leading coefficient

49. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately

$$(0.25 - (-0.25))/2 = 0.25.$$

The period is approximately 1.1.

(c) One model is $y = \frac{1}{4}\cos\left(\frac{2\pi}{1.1}t\right) \approx \frac{1}{4}\cos(5.7t)$

(d)