PART II

CHAPTER P
Preparation for Calculus

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Section P.1 Graphs and Models

Solutions to Even-Numbered Exercises

2. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d)

4. $y = x^3 - x$

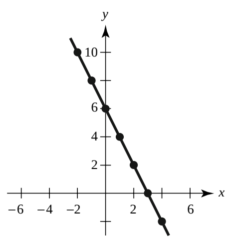
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c)

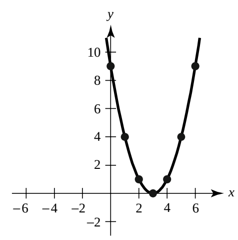
6. $y = 6 - 2x$

x	-2	-1	0	1	2	3	4
y	10	8	6	4	2	0	-2



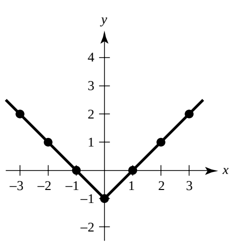
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



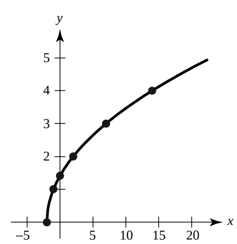
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4

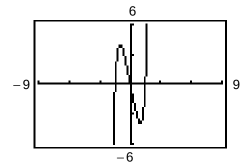


14.

Xmin = -30
Xmax = 30
Xscl = 5
Ymin = -10
Ymax = 40
Yscl = 5

Note that $y = 10$ when $x = 0$ or $x = 10$.

16.



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

18. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0;\ (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2;\ (0, 0), (\pm 2, 0)$

20. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercepts: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1;\ (1, 0)$

22. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0;\ (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3;\ (0, 0), (-3, 0)$

24. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercepts: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3};\ \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

26. $y = x^2 - x$

No symmetry with respect to either axis or the origin.

28. Symmetric with respect to the origin since

$(-y) = (-x)^3 + (-x)$

$-y = -x^3 - x$

$y = x^3 + x.$

30. Symmetric with respect to the x-axis since

$x(-y)^2 = xy^2 = -10.$

32. Symmetric with respect to the origin since

$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$

$xy - \sqrt{4 - x^2} = 0.$

34. $y = \dfrac{x^2}{x^2 + 1}$ is symmetric with respect to the y-axis

since $y = \dfrac{(-x)^2}{(-x)^2 + 1} = \dfrac{x^2}{x^2 + 1}$.

36. $|y| - x = 3$ is symmetric with respect to the x-axis

since $|-y| - x = 3$

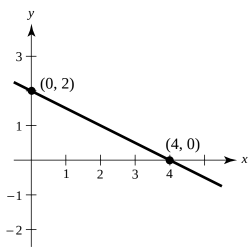
$|y| - x = 3$.

38. $y = -\dfrac{x}{2} + 2$

Intercepts:

$(4, 0), (0, 2)$

Symmetry: none

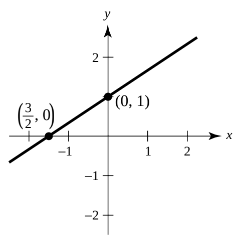


40. $y = \frac{2}{3}x + 1$

Intercepts:

$(0, 1), \left(-\frac{3}{2}, 0\right)$

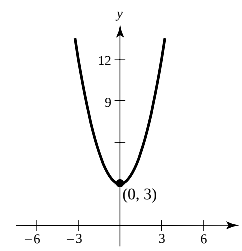
Symmetry: none



42. $y = x^2 + 3$

Intercept: $(0, 3)$

Symmetry: y-axis

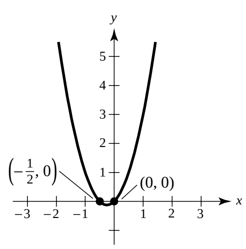


44. $y = 2x^2 + x = x(2x + 1)$

Intercepts:

$(0, 0), \left(-\frac{1}{2}, 0\right)$

Symmetry: none

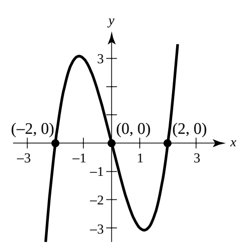


46. $y = x^3 - 4x$

Intercepts:

$(0, 0), (2, 0), (-2, 0)$

Symmetry: origin



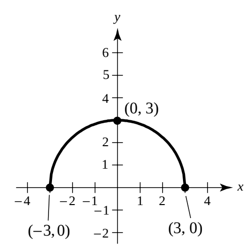
48. $y = \sqrt{9 - x^2}$

Intercepts:

$(-3, 0), (3, 0), (0, 3)$

Symmetry: y-axis

Domain: $[-3, 3]$

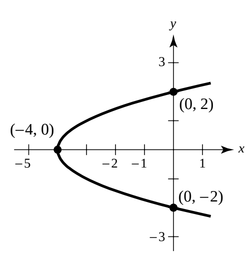


50. $x = y^2 - 4$

Intercepts:

$(0, 2), (0, -2), (-4, 0)$

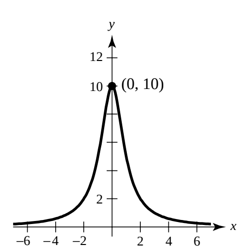
Symmetry: x-axis



52. $y = \dfrac{10}{x^2 + 1}$

Intercepts: $(0, 10)$

Symmetry: y-axis

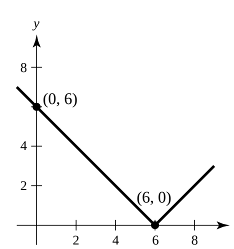


54. $y = |6 - x|$

Intercepts:

$(0, 6), (6, 0)$

Symmetry: none



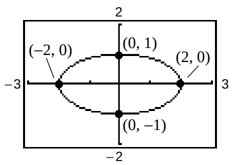
56. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\frac{\sqrt{4 - x^2}}{2}$

Intercepts:

$(-2, 0), (2, 0), (0, -1), (0, 1)$

Symmetry: origin and both axes

Domain: $[-2, 2]$



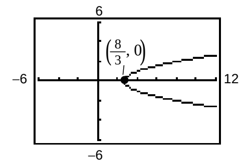
58. $3x - 4y^2 = 8$

$4y^2 = 3x - 8$

$y = \pm\sqrt{\frac{3}{4}x - 2}$

Intercept: $\left(\frac{8}{3}, 0\right)$

Symmetry: x-axis



60. $y = \left(x + \frac{5}{2}\right)(x - 2)\left(x - \frac{3}{2}\right)$ (other answers possible)

62. Some possible equations:

$x = y^2$

$x = |y|$

$x = y^4 + 1$

$x^2 + y^2 = 25$

64. $2x - 3y = 13 \Rightarrow y = \frac{2x - 13}{3}$

$5x + 3y = 1 \Rightarrow y = \frac{1 - 5x}{3}$

$\frac{2x - 13}{3} = \frac{1 - 5x}{3}$

$2x - 13 = 1 - 5x$

$7x = 14$

$x = 2$

The corresponding y-value is $y = -3$.

Point of intersection: $(2, -3)$

66. $5x - 6y = 9 \Rightarrow y = \frac{5x - 9}{6}$

$-7x + 3y = -18 \Rightarrow y = \frac{7x - 18}{3}$

$\frac{5x - 9}{6} = \frac{7x - 18}{3}$

$5x - 9 = 14x - 36$

$27 = 9x$

$x = 3$

The corresponding y-value is $y = 1$.

Point of intersection: $(3, 1)$

68. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$3 - x = (x - 1)^2$

$3 - x = x^2 - 2x + 1$

$0 = x^2 - x - 2 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ and $y = 1$.

Points of intersection: $(-1, -2), (2, 1)$

70. $x^2 + y^2 = 25 \Rightarrow y^2 = 25 - x^2$

$2x + y = 10 \Rightarrow y = 10 - 2x$

$25 - x^2 = (10 - 2x)^2$

$25 - x^2 = 100 - 40x + 4x^2$

$0 = 5x^2 - 40x + 75 = 5(x - 3)(x - 5)$

$x = 3$ or $x = 5$

The corresponding y-values are $y = 4$ and $y = 0$.

Points of intersection: $(3, 4), (5, 0)$

72. $y = x^3 - 4x$

$y = -(x + 2)$

$$x^3 - 4x = -(x + 2)$$

$$x^3 - 3x + 2 = 0$$

$$(x - 1)^2(x + 2) = 0$$

$$x = 1 \quad \text{or} \quad x = -2$$

The corresponding y-values are $y = -3$ and $y = 0$.

Points of intersection: $(1, -3), (-2, 0)$

74. $y = x^4 - 2x^2 + 1$

$y = 1 - x^2$

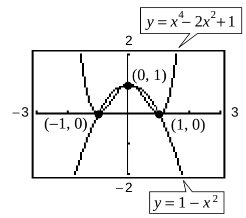
$$1 - x^2 = x^4 - 2x^2 + 1$$

$$0 = x^4 - x^2$$

$$0 = x^2(x + 1)(x - 1)$$

$$x = -1, 0, 1$$

$(-1, 0), (0, 1), (1, 0)$



76. $y = kx + 5$ matches (b).

Use $(1, 7)$: $7 = k(1) + 5 \Longrightarrow k = 2$, thus, $y = 2x + 5$.

$y = x^2 + k$ matches (d).

Use $(1, -9)$: $-9 = (1)^2 + k \Longrightarrow k = -10$, thus, $y = x^2 - 10$.

$y = kx^{3/2}$ matches (a).

Use $(1, 3)$: $3 = k(1)^{3/2} \Longrightarrow k = 3$, thus, $y = 3x^{3/2}$.

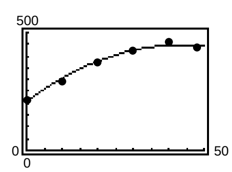
$xy = k$ matches (c).

Use $(1, 36)$: $(1)(36) = k \Longrightarrow k = 36$, thus, $xy = 36$.

78. (a) Using a graphing utility, you obtain

$$y = -0.1283t^2 + 11.0988t + 207.1116$$

(b)



(c) For the year 2004, $t = 54$ and $y \approx 432.3$ acres per farm.

80. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Since $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. Hence, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Since $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

82. True

84. True; the x-intercept is

$$\left(-\frac{b}{2a}, 0\right).$$

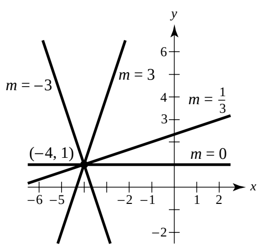
Section P.2 Linear Models and Rates of Change

2. $m = 2$

4. $m = -1$

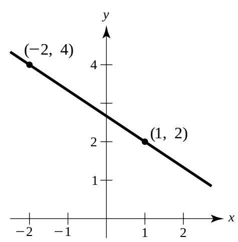
6. $m = \frac{40}{3}$

8.

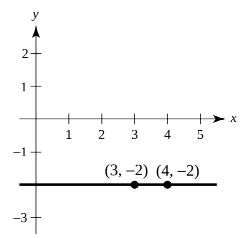


10. $m = \dfrac{4 - 2}{-2 - 1}$

$= -\dfrac{2}{3}$

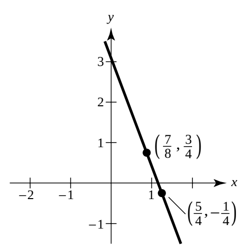


12. $m = \dfrac{-2 - (-2)}{4 - 1} = 0$



14. $m = \dfrac{(3/4) - (-1/4)}{(7/8) - (5/4)}$

$= \dfrac{1}{-3/8} = -\dfrac{8}{3}$



16. Since the slope is undefined, the line is vertical and its equation is $x = -3$. Therefore, three additional points are $(-3, 2)$, $(-3, 3)$, and $(-3, 5)$.

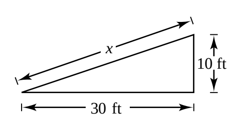
18. The equation of this line is

$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and $(0, 2)$.

20. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$
$$x \approx 31.623 \text{ feet.}$$

22. (a) $m = 400$ indicates that the revenues increase by 400 in one day.

(b) $m = 100$ indicates that the revenues increase by 100 in one day.

(c) $m = 0$ indicates that the revenues do not change from one day to the next.

24. $6x - 5y = 15$

$$y = \frac{6}{5}x - 3$$

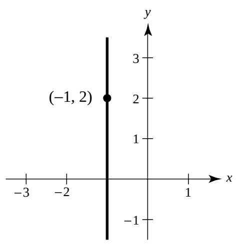
Therefore, the slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

26. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

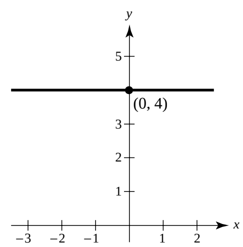
28. $x = -1$

$$x + 1 = 0$$



30. $y = 4$

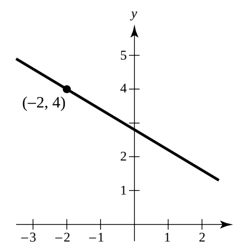
$$y - 4 = 0$$



32. $y - 4 = -\frac{3}{5}(x + 2)$

$$5y - 20 = -3x - 6$$

$$3x + 5y - 14 = 0$$

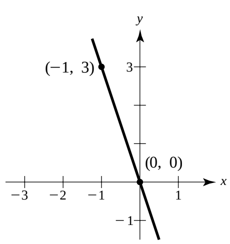


34. $m = \dfrac{3 - 0}{-1 - 0} = -3$

$$y - 0 = -3(x - 0)$$

$$y = -3x$$

$$3x + y = 0$$

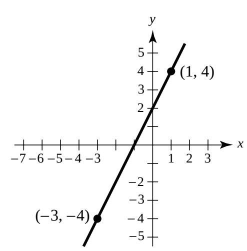


36. $m = \dfrac{4 - (-4)}{1 - (-3)} = \dfrac{8}{4} = 2$

$$y - 4 = 2(x - 1)$$

$$y - 4 = 2x - 2$$

$$0 = 2x - y + 2$$

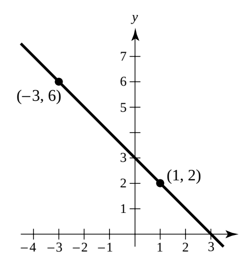


38. $m = \dfrac{6 - 2}{-3 - 1} = \dfrac{4}{-4} = -1$

$$y - 2 = -1(x - 1)$$

$$y - 2 = -x + 1$$

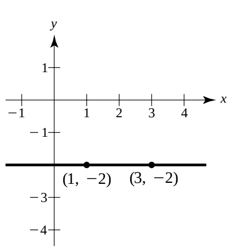
$$x + y - 3 = 0$$



40. $m = 0$

$$y = -2$$

$$y + 2 = 0$$



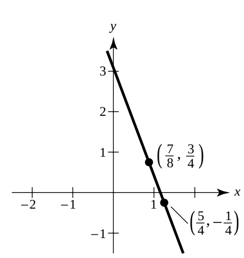
42. $m = \dfrac{(3/4) - (-1/4)}{(7/8) - (5/4)}$

$$= \frac{1}{-3/8} = -\frac{8}{3}$$

$$y + \frac{1}{4} = \frac{-8}{3}\left(x - \frac{5}{4}\right)$$

$$12y + 3 = -32x + 40$$

$$32x + 12y - 37 = 0$$

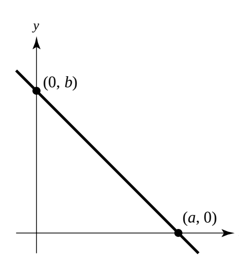


44. $m = -\dfrac{b}{a}$

$$y = \frac{-b}{a}x + b$$

$$\frac{b}{a}x + y = b$$

$$\frac{x}{a} + \frac{y}{b} = 1$$



46. $\dfrac{x}{-2/3} + \dfrac{y}{-2} = 1$

$$\frac{-3x}{2} - \frac{y}{2} = 1$$

$$3x + y = -2$$

$$3x + y + 2 = 0$$

48. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$$\frac{-3}{a} + \frac{4}{a} = 1$$

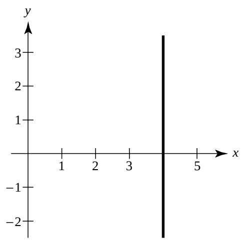
$$\frac{1}{a} = 1$$

$$a = 1 \Rightarrow x + y = 1$$

$$x + y - 1 = 0$$

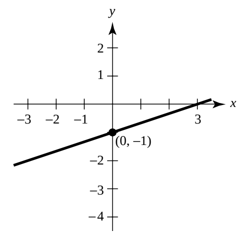
50. $x = 4$

$$x - 4 = 0$$



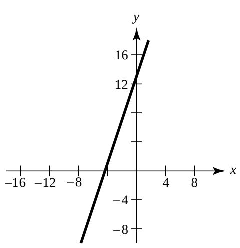
52. $y = \frac{1}{3}x - 1$

$$3y - x + 3 = 0$$



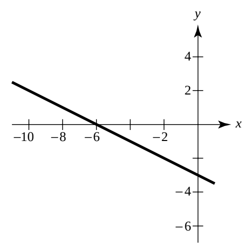
54. $y - 1 = 3(x + 4)$

$$y = 3x + 13$$

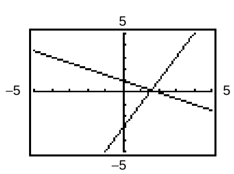


56. $x + 2y + 6 = 0$

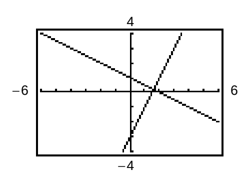
$$y = -\frac{1}{2}x - 3$$



58.



The lines do not appear perpendicular.



The lines appear perpendicular.

The lines are perpendicular because their slopes 2 and $-\frac{1}{2}$ are negative reciprocals of each other. You must use a square setting in order for perpendicular lines to appear perpendicular.

60. $x + y = 7$

$y = -x + 7$

$m = -1$

(a) $y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$x + y + 1 = 0$

(b) $y - 2 = 1(x + 3)$

$y - 2 = x + 3$

$x - y + 5 = 0$

62. $3x + 4y = 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

$m = -\frac{3}{4}$

(a) $y - 4 = -\frac{3}{4}(x + 6)$

$4y - 16 = -3x - 18$

$3x + 4y + 2 = 0$

(b) $y - 4 = \frac{4}{3}(x + 6)$

$3y - 12 = 4x + 24$

$4x - 3y + 36 = 0$

64. (a) $y = 0$

(b) $x = -1 \Longrightarrow x + 1 = 0$

66. The slope is 4.50.

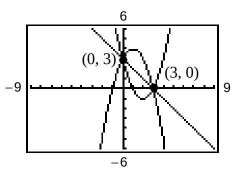
Hence, $V = 4.5(t - 1) + 156$

$= 4.5t + 151.5$

68. The slope is -5600. Hence, $V = -5600(t - 1) + 245{,}000$

$= -5600t + 250{,}600$

70.



You can use the graphing utility to determine that the points of intersection are (0, 3) and (3, 0). Analytically,

$$x^2 - 4x + 3 = -x^2 + 2x + 3$$

$$2x^2 - 6x = 0$$

$$2x(x - 3) = 0$$

$$x = 0 \Longrightarrow y = 3 \Longrightarrow (0, 3)$$

$$x = 3 \Longrightarrow y = 0 \Longrightarrow (3, 0).$$

The slope of the line joining (0, 3) and (3, 0) is $m = (0 - 3)/(3 - 0) = -1$. Hence, an equation of the line is

$$y - 3 = -1(x - 0)$$

$$y = -x + 3.$$

72. $m_1 = \dfrac{-6 - 4}{7 - 0} = -\dfrac{10}{7}$

$m_2 = \dfrac{11 - 4}{-5 - 0} = -\dfrac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

74. Equations of medians:

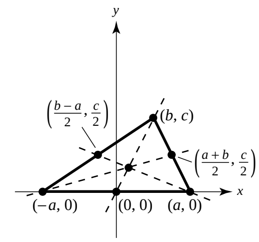
$$y = \frac{c}{b}x$$

$$y = \frac{c}{3a + b}(x + a)$$

$$y = \frac{c}{-3a + b}(x - a)$$

Solving simultaneously, the point of intersection is

$$\left(\frac{b}{3}, \frac{c}{3}\right).$$



76. The slope of the line segment from $\left(\frac{b}{3}, \frac{c}{3}\right)$ to $\left(b, \frac{a^2 - b^2}{c}\right)$ is:

$$m_1 = \frac{[(a^2 - b^2)/c] - (c/3)}{b - (b/3)} = \frac{(3a^2 - 3b^2 - c^2)/(3c)}{(2b)/3} = \frac{3a^2 - 3b^2 - c^2}{2bc}$$

The slope of the line segment from $\left(\frac{b}{3}, \frac{c}{3}\right)$ to $\left(0, \frac{-a^2 + b^2 + c^2}{2c}\right)$ is:

$$m_2 = \frac{[(-a^2 + b^2 + c^2)/(2c)] - (c/3)}{0 - (b/3)} = \frac{(-3a^2 + 3b^2 + 3c^2 - 2c^2)/(6c)}{-b/3} = \frac{3a^2 - 3b^2 - c^2}{2bc}$$

$$m_1 = m_2$$

Therefore, the points are collinear.

78. $C = 0.34x + 150$. If $x = 137$, $C = 0.34(137) + 150 = \$196.58$

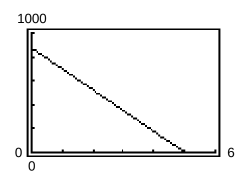
80. (a) Depreciation per year:

$\frac{875}{5} = \$175$

$y = 875 - 175x$

where $0 \le x \le 5$.

(b) $y = 875 - 175(2) = \$525$



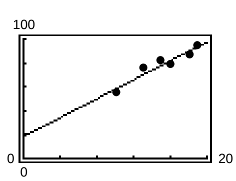
(c) $200 = 875 - 175x$

$175x = 675$

$x \approx 3.86$ years

82. (a) $y = 18.91 + 3.97x$ (x = quiz score, y = test score)

(b)



(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

84. $4x + 3y - 10 = 0 \Rightarrow d = \frac{|4(2) + 3(3) - 10|}{\sqrt{4^2 + 3^2}} = \frac{7}{5}$

86. $x + 1 = 0 \Rightarrow d = \frac{|1(6) + (0)(2) + 1|}{\sqrt{1^2 + 0^2}} = 7$

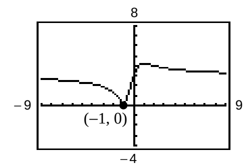
88. A point on the line $3x - 4y = 1$ is $(-1, -1)$. The distance from the point $(-1, -1)$ to $3x - 4y - 10 = 0$ is

$$d = \frac{|-3 + 4 - 10|}{5} = \frac{9}{5}.$$

90. $y = mx + 4 \Longrightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}}$$

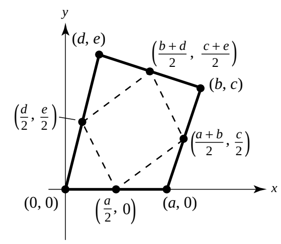
$$= \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$



The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point $(3, 1)$.

92. For simplicity, let the vertices of the quadrilateral be $(0, 0)$, $(a, 0)$, (b, c), and (d, e), as shown in the figure. The midpoints of the sides are

$$\left(\frac{a}{2}, 0\right), \left(\frac{a+b}{2}, \frac{c}{2}\right), \left(\frac{b+d}{2}, \frac{c+e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$



The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a+b}{2} - \frac{a}{2}} = \frac{\frac{c+e}{2} - \frac{e}{2}}{\frac{b+d}{2} - \frac{d}{2}} = \frac{c}{b}$$

$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c+e}{2}}{\frac{a+b}{2} - \frac{b+d}{2}} = -\frac{e}{a-d}$$

Therefore, the figure is a paralleogram.

94. If $m_1 = -1/m_2$, then $m_1m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1m_3 = -1$. Hence, $m_2 = m_3 \Longrightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

96. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

Section P.3 Functions and Their Graphs

2. (a) $f(-2) = \sqrt{-2 + 3} = \sqrt{1} = 1$

(b) $f(6) = \sqrt{6 + 3} = \sqrt{9} = 3$

(c) $f(c) = \sqrt{c + 3}$

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 3}$

4. (a) $g(4) = 4^2(4 - 4) = 0$

(b) $g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $g(c) = c^2(c - 4) = c^3 - 4c^2$

(d) $g(t + 4) = (t + 4)^2(t + 4 - 4)$

$= (t + 4)^2 t = t^3 + 8t^2 + 16t$

6. (a) $f(\pi) = \sin\pi = 0$

(b) $f\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

(c) $f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

8. $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{3x - 1 - (3 - 1)}{x - 1} = \dfrac{3(x - 1)}{x - 1} = 3, x \neq 1$

10. $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{x^3 - x - 0}{x - 1} = \dfrac{x(x + 1)(x - 1)}{x - 1} = x(x + 1), x \neq 1$

12. $g(x) = x^2 - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

14. $h(t) = \cot t$

Domain: all $t \neq k\pi$, k an integer

Range: $(-\infty, \infty)$

16. $g(x) = \dfrac{2}{x - 1}$

Domain: $(-\infty, 1), (1, \infty)$

Range: $(-\infty, 0), (0, \infty)$

18. $f(x) = \begin{cases} x^2 + 2, x \le 1 \\ 2x^2 + 2, x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(s^2 + 2) = 2(s^2 + 2)^2 = 2s^4 + 8s^2 + 10$

(Note: $s^2 + 2 > 1$ for all s)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

20. $f(x) = \begin{cases} \sqrt{x + 4}, x \le 5 \\ (x - 5)^2, x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3 + 4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0 + 4} = 2$

(c) $f(5) = \sqrt{5 + 4} = 3$

(d) $f(10) = (10 - 5)^2 = 25$

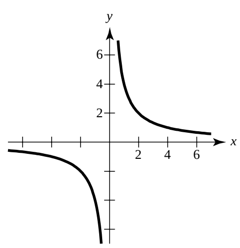
Domain: $[-4, \infty)$

Range: $[0, \infty)$

22. $g(x) = \dfrac{4}{x}$

Domain: $(-\infty, 0), (0, \infty)$

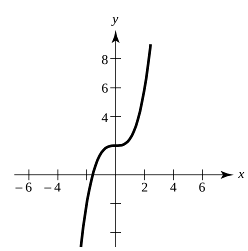
Range: $(-\infty, 0), (0, \infty)$



24. $f(x) = \frac{1}{2}x^3 + 2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$



26. $f(x) = x + \sqrt{4 - x^2}$

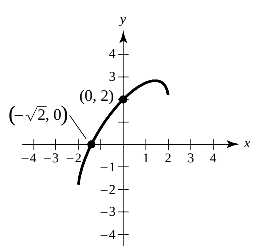
Domain: $[-2, 2]$

Range:

$[-2, 2\sqrt{2}] \approx [-2, 2.83]$

y-intercept: $(0, 2)$

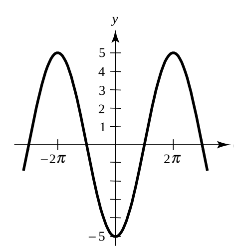
x-intercept: $\left(-\sqrt{2}, 0\right)$



28. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

Domain: $(-\infty, \infty)$

Range: $[-5, 5]$



30. $\sqrt{x^2 - 4} - y = 0 \Rightarrow y = \sqrt{x^2 - 4}$

y is a function of x. Vertical lines intersect the graph at most once.

32. $x^2 + y^2 = 4$

$y = \pm\sqrt{4 - x^2}$

y is not a function of x. Some vertical lines intersect the graph twice.

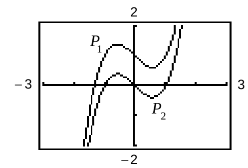
34. $x^2 + y = 4 \Rightarrow y = 4 - x^2$

y is a function of x since there is one value of y for each x.

36. $x^2y - x^2 + 4y = 0 \Rightarrow y = \dfrac{x^2}{x^2 + 4}$

y is a function of x since there is one value of y for each x.

38. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, we have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Since the graph has no breaks, the graph must cross the x-axis at least one time.

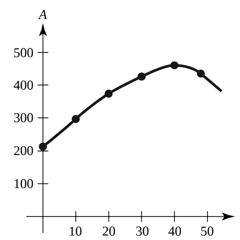


40. The function is $f(x) = cx$. Since $(1, 1/4)$ satisfies the equation, $c = 1/4$. Thus, $f(x) = (1/4)x$.

42. The function is $h(x) = c\sqrt{|x|}$. Since $(1, 3)$ satisfies the equation, $c = 3$. Thus, $h(x) = 3\sqrt{|x|}$.

44. The student travels $\dfrac{2 - 0}{4 - 0} = \dfrac{1}{2}$ mi/min during the first 4 minutes. The student is stationary for the following 2 minutes. Finally, the student travels $\dfrac{6 - 2}{10 - 6} = 1$ mi/min during the final 4 minutes.

46. (a)



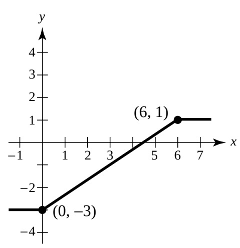
(b) $A(15) \approx 345$ acres/farm

48. (a) $g(x) = f(x - 4)$

$g(6) = f(2) = 1$

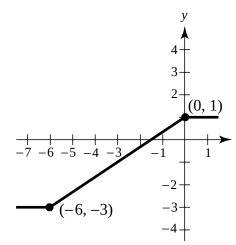
$g(0) = f(-4) = -3$

Shift f right 4 units



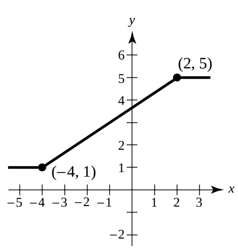
(b) $g(x) = f(x + 2)$

Shift f left 2 units



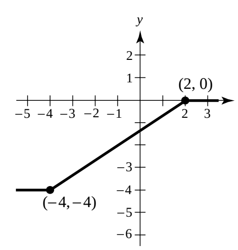
(c) $g(x) = f(x) + 4$

Vertical shift upwards 4 units



(d) $g(x) = f(x) - 1$

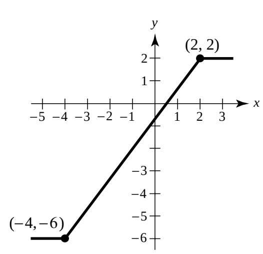
Vertical shift down 1 unit



(e) $g(x) = 2f(x)$

$g(2) = 2f(2) = 2$

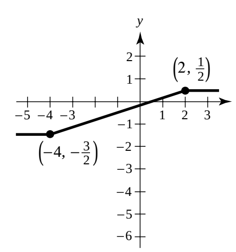
$g(-4) = 2f(-4) = -6$



(f) $g(x) = \frac{1}{2}f(x)$

$g(2) = \frac{1}{2}f(2) = \frac{1}{2}$

$g(-4) = \frac{1}{2}f(-4) = -\frac{3}{2}$



50. (a) $h(x) = \sin(x + (\pi/2)) + 1$ is a horizontal shift $\pi/2$ units to the left, followed by a vertical shift 1 unit upwards.

(b) $h(x) = -\sin(x - 1)$ is a horizontal shift 1 unit to the right followed by a reflection about the x-axis.

52. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1 \quad (x \geq 0)$

54. $f(x) = x^2 - 1, g(x) = \cos x$

$(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^2 x - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = \cos(x^2 - 1)$

Domain: $(-\infty, \infty)$

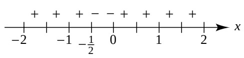
No, $f \circ g \neq g \circ f$.

56. $(f \circ g)(x) = f(\sqrt{x + 2}) = \dfrac{1}{\sqrt{x + 2}}$

Domain: $(-2, \infty)$

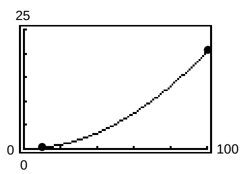
$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = \sqrt{\dfrac{1}{x} + 2} = \sqrt{\dfrac{1 + 2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right], (0, \infty)$

58. (a)



(b) $H(1.6x) = 0.002(1.6x)^2 + 0.005(1.6x) - 0.029$

$= 0.00512x^2 + 0.008x - 0.029$

60. $f(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -f(x)$

Odd

62. $f(-x) = \sin^2(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x$

Even

64. (a) If f is even, then $(-4, 9)$ is on the graph.

(b) If f is odd, then $(-4, -9)$ is on the graph.

66. $f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$

$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$= f(x)$

Even

68. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then

$F(-x) = f(-x)g(-x) = f(x)[-g(x)] = -f(x)g(x) = -F(x).$

Thus, $F(x)$ is odd.

70. (a) Let $F(x) = f(x) \pm g(x)$ where f and g are even. Then, $F(-x) = f(-x) \pm g(-x) = f(x) \pm g(x) = F(x)$. Thus, $F(x)$ is even.

(b) Let $F(x) = f(x) \pm g(x)$ where f and g are odd. Then, $F(-x) = f(-x) \pm g(-x) = -f(x) \mp g(x) = -F(x)$. Thus, $F(x)$ is odd.

(c) Let $F(x) = f(x) \pm g(x)$ where f is odd and g is even. Then, $F(-x) = f(-x) \pm g(-x) = -f(x) \pm g(x)$. Thus, $F(x)$ is neither odd nor even.

72. By equating slopes, $\dfrac{y-2}{0-3} = \dfrac{0-2}{x-3}$

$$y - 2 = \frac{6}{x-3}$$

$$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3},$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

74. True

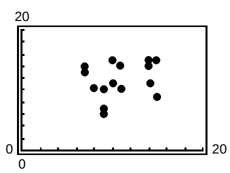
76. False; let $f(x) = x^2$. Then $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. Thus, $3f(x) \neq f(3x)$.

Section P.4 Fitting Models to Data

2. Trigonometric function

4. No relationship

6. (a)

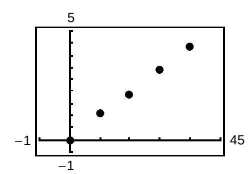


No, the relationship does not appear to be linear.

(b) Quiz scores are dependent on several variables such as study time, class attendance, etc. These variables may change from one quiz to the next.

8. (a) $s = 9.7t + 0.4$

(b)

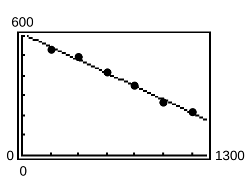


The model fits well.

(c) If $t = 2.5$, $s = 24.65$ meters/second.

10. (a) Linear model: $H = -0.3323t + 612.9333$

(b)



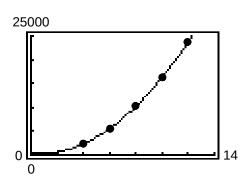
The fit is very good.

(c) When $t = 500$,

$$H = -0.3323(500) + 612.9333 \approx 446.78.$$

12. (a) $S = 180.89x^2 - 205.79x + 272$

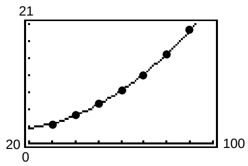
(b)



(c) When $x = 2$, $S \approx 583.98$ pounds.

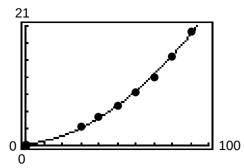
14. (a) $t = 0.00271s^2 - 0.0529s + 2.671$

(b)



(c) The curve levels off for $s < 20$.

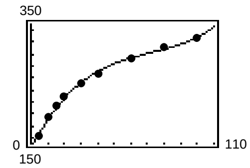
(d) $t = 0.002s^2 + 0.0346s + 0.183$



The model is better for low speeds.

16. (a) $T = 2.9856 \times 10^{-4} p^3 - 0.0641 p^2 + 5.2826p + 143.1$

(b)



(c) For $T = 300°\text{F}$, $p \approx 68.29$ pounds per square inch.

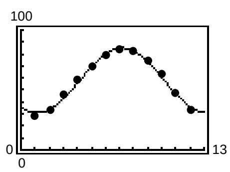
(d) The model is based on data up to 100 pounds per square inch.

18. (a) $H(t) = 84.4 + 4.28 \sin\left(\dfrac{\pi t}{6} + 3.86\right)$

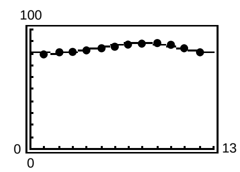
One model is

$$C(t) = 58 + 27 \sin\left(\frac{\pi t}{6} + 4.1\right).$$

(b)



(c)



(d) The average in Honolulu is 84.4.

The average in Chicago is 58.

(e) The period is 12 months (1 year).

(f) Chicago has greater variability $(27 > 4.28)$.

20. Answers will vary.

Review Exercises for Chapter P

2. $y = (x - 1)(x - 3)$

$x = 0 \Longrightarrow y = (0 - 1)(0 - 3) = 3 \Longrightarrow (0, 3)$ y-intercept

$y = 0 \Longrightarrow 0 = (x - 1)(x - 3) \Longrightarrow x = 1, 3 \Longrightarrow (1, 0), (3, 0)$ x-intercepts

4. $xy = 4$

$x = 0$ and $y = 0$ are both impossible. No intercepts.

6. Symmetric with respect to y-axis since

$y = (-x)^4 - (-x)^2 + 3$

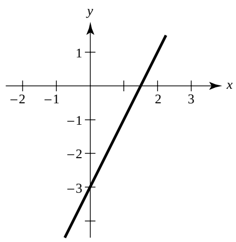
$y = x^4 - x^2 + 3.$

8. $4x - 2y = 6$

$$y = 2x - 3$$

Slope: 2

y-intercept: -3



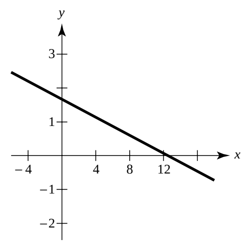
10. $0.02x + 0.15y = 0.25$

$$2x + 15y = 25$$

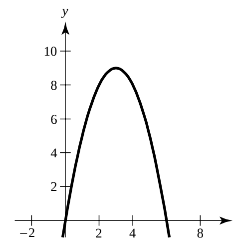
$$y = -\frac{2}{15}x + \frac{5}{3}$$

Slope: $-\frac{2}{15}$

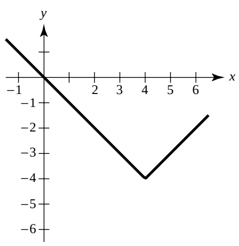
y-intercept: $\frac{5}{3}$



12. $y = x(6 - x)$



14. $y = |x - 4| - 4$



16. $y = 8\sqrt[3]{x - 6}$

Xmin = -40
Xmax = 40
Xscl = 10
Ymin = -40
Ymax = 40
Yscl = 10

18.

$$y = x + 1$$

$$(x + 1) - x^2 = 7$$

$$0 = x^2 - x + 6$$

No real solution
No points of intersection
The graphs of $y = x + 1$ and $y = x^2 + 7$ do not intersect.

20. $y = kx^3$

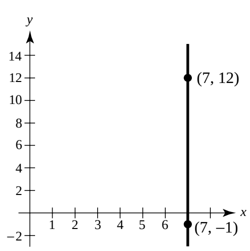
(a) $4 = k(1)^3 \Longrightarrow k = 4$ and $y = 4x^3$

(b) $1 = k(-2)^3 \Longrightarrow k = -\frac{1}{8}$ and $y = -\frac{1}{8}x^3$

(c) $0 = k(0)^3 \Longrightarrow$ any k will do!

(d) $-1 = k(-1)^3 \Longrightarrow k = 1 \Longrightarrow y = x^3$

22.



The line is vertical and has no slope.

24.

$$\frac{3 - (-1)}{-3 - t} = \frac{3 - 6}{-3 - 8}$$

$$\frac{4}{-3 - t} = \frac{-3}{-11}$$

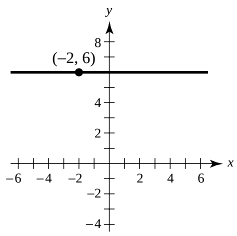
$$-44 = 9 + 3t$$

$$-53 = 3t$$

$$t = -\frac{53}{3}$$

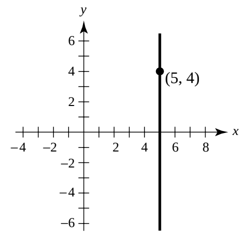
26. $y - 6 = 0(x - (-2))$

$y = 6$ Horizontal line



28. m is undefined. Line is vertical.

$x = 5$



30. (a) $y - 3 = -\frac{2}{3}(x - 1)$

$3y - 9 = -2x + 2$

$2x + 3y - 11 = 0$

(b) Slope of perpendicular line is 1.

$y - 3 = 1(x - 1)$

$y = x + 2$

$0 = x - y + 2$

(c) $m = \frac{4 - 3}{2 - 1} = 1$

$y - 3 = 1(x - 1)$

$y = x + 2$

$0 = x - y + 2$

(d) $y = 3$

$y - 3 = 0$

32. (a) $C = 9.25t + 13.50t + 36{,}500$

$= 22.75t + 36{,}500$

(b) $R = 30t$

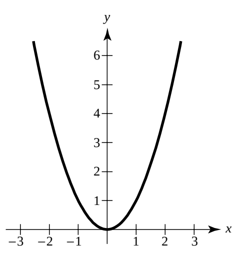
(c) $30t = 22.75t + 36{,}500$

$7.25t = 36{,}000$

$t \approx 5034.48$ hours to break even.

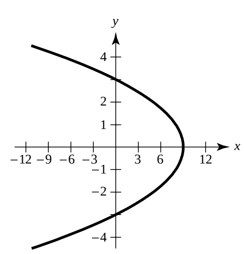
34. $x^2 - y = 0$

Function of x since there is one value for y for each x.



36. $x = 9 - y^2$

Not a function of x since there are two values of y for some x.



38. (a) $f(-4) = (-4)^2 + 2 = 18$ (because $-4 < 0$)

(b) $f(0) = |0 - 2| = 2$

(c) $f(1) = |1 - 2| = 1$

40. $f(x) = 1 - x^2$ and $g(x) = 2x + 1$

(a) $f(x) - g(x) = (1 - x^2) - (2x + 1) = -x^2 - 2x$

(b) $f(x)g(x) = (1 - x^2)(2x + 1) = -2x^3 - x^2 + 2x + 1$

(c) $g(f(x)) = g(1 - x^2) = 2(1 - x^2) + 1 = 3 - 2x^2$