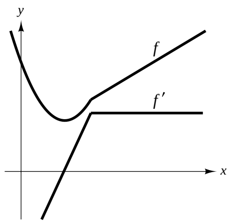
Review Exercises for Chapter 4

1.



3. $\int (2x^2 + x - 1)\, dx = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$

5. $\int \frac{x^3 + 1}{x^2}\, dx = \int \left(x + \frac{1}{x^2}\right) dx = \frac{1}{2}x^2 - \frac{1}{x} + C$

7. $\int (4x - 3\sin x)\, dx = 2x^2 + 3\cos x + C$

9. $f'(x) = -2x, \ (-1, 1)$

$f(x) = \int -2x\, dx = -x^2 + C$

When $x = -1$:

$y = -1 + C = 1$

$C = 2$

$y = 2 - x^2$

11. $a(t) = a$

$v(t) = \int a\, dt = at + C_1$

$v(0) = 0 + C_1 = 0$ when $C_1 = 0$.

$v(t) = at$

$s(t) = \int at\, dt = \frac{a}{2}t^2 + C_2$

$s(0) = 0 + C_2 = 0$ when $C_2 = 0$.

$s(t) = \frac{a}{2}t^2$

$s(30) = \frac{a}{2}(30)^2 = 3600$ or

$a = \frac{2(3600)}{(30)^2} = 8$ ft/sec².

$v(30) = 8(30) = 240$ ft/sec

13. $a(t) = -32$

$v(t) = -32t + 96$

$s(t) = -16t^2 + 96t$

(a) $v(t) = -32t + 96 = 0$ when $t = 3$ sec.

(b) $s(3) = -144 + 288 = 144$ ft

(c) $v(t) = -32t + 96 = \frac{96}{2}$ when $t = \frac{3}{2}$ sec.

(d) $s\left(\frac{3}{2}\right) = -16\left(\frac{9}{4}\right) + 96\left(\frac{3}{2}\right) = 108$ ft

15. (a) $\sum_{i=1}^{10} (2i - 1)$

(b) $\sum_{i=1}^{n} i^3$

(c) $\sum_{i=1}^{10} (4i + 2)$

17. $y = \dfrac{10}{x^2 + 1}, \Delta x = \dfrac{1}{2}, n = 4$

$$S(n) = S(4) = \frac{1}{2}\left[\frac{10}{1} + \frac{10}{(1/2)^2 + 1} + \frac{10}{(1)^2 + 1} + \frac{10}{(3/2)^2 + 1}\right]$$

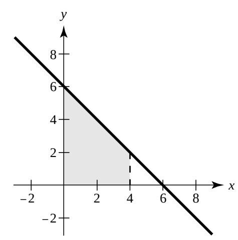
$$\approx 13.0385$$

$$s(n) = s(4) = \frac{1}{2}\left[\frac{10}{(1/2)^2 + 1} + \frac{10}{1 + 1} + \frac{10}{(3/2)^2 + 1} + \frac{10}{2^2 + 1}\right]$$

$$\approx 9.0385$$

$9.0385 < \text{Area of Region} < 13.0385$

19. $y = 6 - x, \Delta x = \dfrac{4}{n}$, right endpoints



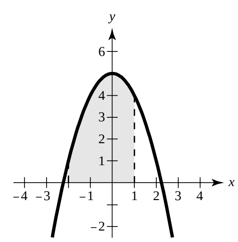
$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(ci)\,\Delta x$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(6 - \frac{4i}{n}\right)\frac{4}{n}$$

$$= \lim_{n \to \infty} \frac{4}{n}\left[6n - \frac{4}{n}\frac{n(n + 1)}{2}\right]$$

$$= \lim_{n \to \infty} \left[24 - 8\,\frac{n + 1}{n}\right] = 24 - 8 = 16$$

21. $y = 5 - x^2, \Delta x = \dfrac{3}{n}$



$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(ci)\,\Delta x$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left[5 - \left(-2 + \frac{3i}{n}\right)^2\right]\left(\frac{3}{n}\right)$$

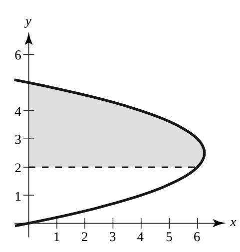
$$= \lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} \left[1 + \frac{12i}{n} - \frac{9i^2}{n^2}\right]$$

$$= \lim_{n \to \infty} \frac{3}{n}\left[n + \frac{12}{n}\frac{n(n + 1)}{2} - \frac{9}{n^2}\frac{n(n + 1)(2n + 1)}{6}\right]$$

$$= \lim_{n \to \infty} \left[3 + 18\,\frac{n + 1}{n} - \frac{9}{2}\frac{(n + 1)(2n + 1)}{n^2}\right]$$

$$= 3 + 18 - 9 = 12$$

23. $x = 5y - y^2, 2 \le y \le 5, \Delta y = \dfrac{3}{n}$



$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} \left[5\left(2 + \frac{3i}{n}\right) - \left(2 + \frac{3i}{n}\right)^2\right]\left(\frac{3}{n}\right)$$

$$= \lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} \left[10 + \frac{15i}{n} - 4 - 12\frac{i}{n} - \frac{9i^2}{n^2}\right]$$

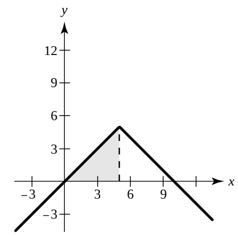
$$= \lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} \left[6 + \frac{3i}{n} - \frac{9i^2}{n^2}\right]$$

$$= \lim_{n \to \infty} \frac{3}{n}\left[6n + \frac{3}{n}\frac{n(n + 1)}{2} - \frac{9}{n^2}\frac{n(n + 1)(2n + 1)}{6}\right]$$

$$= \left[18 + \frac{9}{2} - 9\right] = \frac{27}{2}$$

25. $\lim_{\|\Delta\| \quad \infty} \sum_{i=1}^{n} (2ci - 3)\, \Delta xi = \int_4^6 (2x - 3)\, dx$

27.



$\int_0^5 \Big(5 - |x - 5|\, dx = \int_0^5 (5 - (5 - x))\, dx = \int_0^5 x\, dx = \frac{25}{2}$

(triangle)

29. (a) $\int_2^6 [f(x) + g(x)]\, dx = \int_2^6 f(x)\, dx + \int_2^6 g(x)\, dx = 10 + 3 = 13$

(b) $\int_2^6 [f(x) - g(x)]\, dx = \int_2^6 f(x)\, dx - \int_2^6 g(x)\, dx = 10 - 3 = 7$

(c) $\int_2^6 [2f(x) - 3g(x)]\, dx = 2\int_2^6 f(x)\, dx - 3\int_2^6 g(x)\, dx = 2(10) - 3(3) = 11$

(d) $\int_2^6 5f(x)\, dx = 5\int_2^6 f(x)\, dx = 5(10) = 50$

31. $\int_1^8 \left(\sqrt[3]{x} + 1\right) dx = \left[\frac{3}{4}x^{4/3} + x\right]_1^8 = \left[\frac{3}{4}(16) + 8\right] - \left[\frac{3}{4} + 1\right] = \frac{73}{4}$ (c)

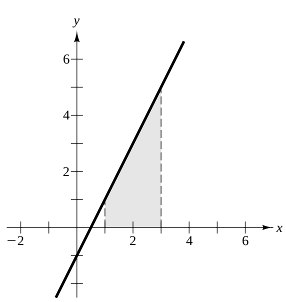
33. $\int_0^4 (2 + x)\, dx = \left[2x + \frac{x^2}{2}\right]_0^4 = 8 + \frac{16}{2} = 16$

35. $\int_{-1}^{1} (4t^3 - 2t)\, dt = \left[t^4 - t^2\right]_{-1}^{1} = 0$

37. $\int_4^9 x\sqrt{x}\, dx = \int_4^9 x^{3/2}\, dx = \left[\frac{2}{5}x^{5/2}\right]_4^9 = \frac{2}{5}\left[\left(\sqrt{9}\right)^5 - \left(\sqrt{4}\right)^5\right] = \frac{2}{5}(243 - 32) = \frac{422}{5}$

39. $\int_0^{3\pi/4} \sin\theta\, d\theta = \Big[-\cos\theta\Big]_0^{3\pi/4} = -\left(-\frac{\sqrt{2}}{2}\right) + 1 = 1 + \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + 2}{2}$

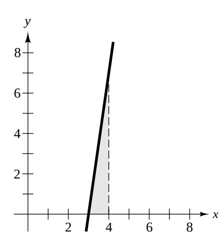
41. $\int_1^3 (2x - 1)\, dx = \left[x^2 - x\right]_1^3 = 6$



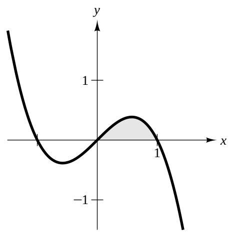
43. $\int_3^4 (x^2 - 9)\, dx = \left[\frac{x^3}{3} - 9x\right]_3^4$

$= \left(\frac{64}{3} - 36\right) - (9 - 27)$

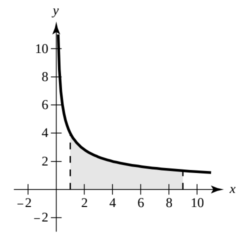
$= \frac{64}{3} - \frac{54}{3} = \frac{10}{3}$



45. $\int_0^1 (x - x^3)\,dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$



47. Area $= \int_1^9 \frac{4}{\sqrt{x}}\,dx = \left[\frac{4x^{1/2}}{(1/2)}\right]_1^9 = 8(3 - 1) = 16$

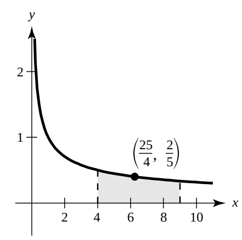


49. $\frac{1}{9-4}\int_4^9 \frac{1}{\sqrt{x}}\,dx = \left[\frac{1}{5}2\sqrt{x}\right]_4^9 = \frac{2}{5}(3 - 2) = \frac{2}{5}$ Average value

$\frac{2}{5} = \frac{1}{\sqrt{x}}$

$\sqrt{x} = \frac{5}{2}$

$x = \frac{25}{4}$



51. $F'(x) = x^2\sqrt{1 + x^3}$

53. $F'(x) = x^2 + 3x + 2$

55. $\int (x^2 + 1)^3\,dx = \int (x^6 + 3x^4 + 3x^2 + 1)\,dx = \frac{x^7}{7} + \frac{3}{5}x^5 + x^3 + x + C$

57. $u = x^3 + 3$, $du = 3x^2\,dx$

$\int \frac{x^2}{\sqrt{x^3 + 3}}\,dx = \int (x^3 + 3)^{-1/2}\,x^2\,dx = \frac{1}{3}\int (x^3 + 3)^{-1/2}\,3x^2\,dx = \frac{2}{3}(x^3 + 3)^{1/2} + C$

59. $u = 1 - 3x^2$, $du = -6x\,dx$

$\int x(1 - 3x^2)^4\,dx = -\frac{1}{6}\int (1 - 3x^2)^4(-6x\,dx) = -\frac{1}{30}(1 - 3x^2)^5 + C = \frac{1}{30}(3x^2 - 1)^5 + C$

61. $\int \sin^3 x \cos x\,dx = \frac{1}{4}\sin^4 x + C$

63. $\int \frac{\sin\theta}{\sqrt{1 - \cos\theta}}\,dx = \int (1 - \cos\theta)^{-1/2}\sin\theta\,d\theta = 2(1 - \cos\theta)^{1/2} + C = 2\sqrt{1 - \cos\theta} + C$

65. $\int \tan^n x \sec^2 x\,dx = \frac{\tan^{n+1} x}{n + 1} + C, n \neq -1$

67. $\int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x\,dx = \frac{1}{\pi}\int (1 + \sec \pi x)^2(\pi \sec \pi x \tan \pi x)\,dx = \frac{1}{3\pi}(1 + \sec \pi x)^3 + C$

69. $\int_{-1}^2 x(x^2 - 4)\,dx = \frac{1}{2}\int_{-1}^2 (x^2 - 4)(2x)\,dx = \frac{1}{2}\left[\frac{(x^2 - 4)^2}{2}\right]_{-1}^2 = \frac{1}{4}[0 - 9] = -\frac{9}{4}$

71. $\displaystyle\int_0^3 \frac{1}{\sqrt{1+x}}\,dx = \int_0^3 (1+x)^{-1/2}\,dx = \Big[2(1+x)^{1/2}\Big]_0^3 = 4 - 2 = 2$

73. $u = 1 - y,\ y = 1 - u,\ dy = -du$

When $y = 0$, $u = 1$. When $y = 1$, $u = 0$.

$$2\pi\int_0^1 (y+1)\sqrt{1-y}\,dy = 2\pi\int_1^0 -[(1-u)+1]\sqrt{u}\,du$$

$$= 2\pi\int_1^0 (u^{3/2} - 2u^{1/2})\,du = 2\pi\Big[\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2}\Big]_1^0 = \frac{28\pi}{15}$$

75. $\displaystyle\int_0^\pi \cos\Big(\frac{x}{2}\Big)\,dx = 2\int_0^\pi \cos\Big(\frac{x}{2}\Big)\frac{1}{2}\,dx = \Big[2\sin\Big(\frac{x}{2}\Big)\Big]_0^\pi = 2$

77. $u = 1 - x,\ x = 1 - u,\ dx = -du$

When $x = a$, $u = 1 - a$. When $x = b$, $u = 1 - b$.

$$P_{a,b} = \int_a^b \frac{15}{4}x\sqrt{1-x}\,dx = \frac{15}{4}\int_{1-a}^{1-b} -(1-u)\sqrt{u}\,du$$

$$= \frac{15}{4}\int_{1-a}^{1-b}(u^{3/2} - u^{1/2})\,du = \frac{15}{4}\Big[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big]_{1-a}^{1-b} = \frac{15}{4}\Big[\frac{2u^{3/2}}{15}(3u-5)\Big]_{1-a}^{1-b} = \Big[-\frac{(1-x)^{3/2}}{2}(3x+2)\Big]_a^b$$

(a) $\displaystyle P_{0.50,\,0.75} = \Big[-\frac{(1-x)^{3/2}}{2}(3x+2)\Big]_{0.50}^{0.75} = 0.353 = 35.3\%$

(b) $\displaystyle P_{0,\,b} = \Big[-\frac{(1-x)^{3/2}}{2}(3x+2)\Big]_0^b = -\frac{(1-b)^{3/2}}{2}(3b+2) + 1 = 0.5$

$(1-b)^{3/2}(3b+2) = 1$

$b \approx 0.586 = 58.6\%$

79. $p = 1.20 + 0.04t$

$$C = \frac{15{,}000}{M}\int_t^{t+1} p\,ds$$

(a) 2000 corresponds to $t = 10$.

$$C = \frac{15{,}000}{M}\int_{10}^{11}[1.20 + 0.04t]\,dt$$

$$= \frac{15{,}000}{M}\Big[1.20t + 0.02t^2\Big]_{10}^{11} = \frac{24{,}300}{M}$$

(b) 2005 corresponds to $t = 15$.

$$C = \frac{15{,}000}{M}\Big[1.20t + 0.02t^2\Big]_{15}^{16} = \frac{27{,}300}{M}$$

81. Trapezoidal Rule ($n = 4$): $\displaystyle\int_1^2 \frac{1}{1+x^3}\,dx \approx \frac{1}{8}\Big[\frac{1}{1+1^3} + \frac{2}{1+(1.25)^3} + \frac{2}{1+(1.5)^3} + \frac{2}{1+(1.75)^3} + \frac{1}{1+2^3}\Big] \approx 0.257$

Simpson's Rule ($n = 4$): $\displaystyle\int_1^2 \frac{1}{1+x^3}\,dx \approx \frac{1}{12}\Big[\frac{1}{1+1^3} + \frac{4}{1+(1.25)^3} + \frac{2}{1+(1.5)^3} + \frac{4}{1+(1.75)^3} + \frac{1}{1+2^3}\Big] \approx 0.254$

Graphing utility: 0.254

83. Trapezoidal Rule ($n = 4$): $\displaystyle\int_0^{\pi/2} \sqrt{x}\cos x\,dx \approx 0.637$

Simpson's Rule ($n = 4$): 0.685

Graphing Utility: 0.704

85. (a) $R < I < T < L$

(b) $\displaystyle S(4) = \frac{4-0}{3(4)}[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)]$

$$\approx \frac{1}{3}\Big[4 + 4(2) + 2(1) + 4\Big(\frac{1}{2}\Big) + \frac{1}{4}\Big] \approx 5.417$$

Problem Solving for Chapter 4

1. (a) $L(1) = \int_1^1 \frac{1}{t}\,dt = 0$

(b) $L'(x) = \frac{1}{x}$ by the Second Fundamental Theorem of Calculus.

$L'(1) = 1$

(c) $L(x) = 1 = \int_1^x \frac{1}{t}\,dt$ for $x \approx 2.718$

$\int_1^{2.718} \frac{1}{t}\,dt = 0.999896$ (**Note:** The exact value of x is e, the base of the natural logarithm function.)

(d) We first show that $\int_1^{x_1} \frac{1}{t}\,dt = \int_{1/x_1}^{1} \frac{1}{t}\,dt$.

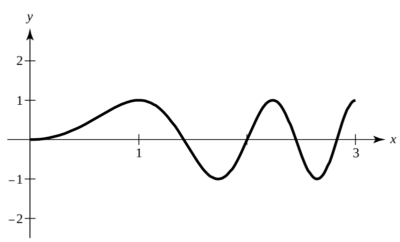
To see this, let $u = \frac{t}{x_1}$ and $du = \frac{1}{x_1}\,dt$.

Then $\int_1^{x_1} \frac{1}{t}\,dt = \int_{1/x_1}^{1} \frac{1}{ux_1}(x_1\,du) = \int_{1/x_1}^{1} \frac{1}{u}\,du = \int_{1/x_1}^{1} \frac{1}{t}\,dt$.

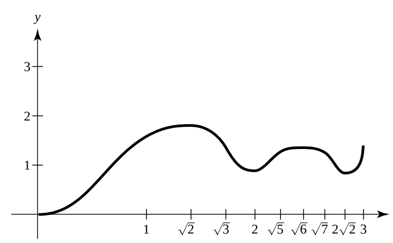
Now,
$$\begin{aligned} L(x_1x_2) &= \int_1^{x_1x_2} \frac{1}{t}\,dt = \int_{1/x_1}^{x_2} \frac{1}{u}\,du \left(\text{using } u = \frac{t}{x_1}\right) \\ &= \int_{1/x_1}^{1} \frac{1}{u}\,du + \int_1^{x_2} \frac{1}{u}\,du \\ &= \int_1^{x_1} \frac{1}{u}\,du + \int_1^{x_2} \frac{1}{u}\,du \\ &= L(x_1) + L(x_2). \end{aligned}$$

3. $S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt$

(a)



(b)



The zeros of $y = \sin\frac{\pi x^2}{2}$ correspond to the relative extrema of $S(x)$.

(c) $S'(x) = \sin\frac{\pi x^2}{2} = 0 \qquad \frac{\pi x^2}{2} = n\pi \qquad x^2 = 2n \qquad x = \sqrt{2n}$, n integer.

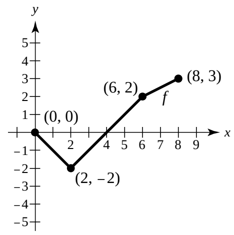
Relative maximum at $x = \sqrt{2} \approx 1.4142$ and $x = \sqrt{6} \approx 2.4495$

Relative minimum at $x = 2$ and $x = \sqrt{8} \approx 2.8284$

(d) $S''(x) = \cos\left(\frac{\pi x^2}{2}\right)(\pi x) = 0 \qquad \frac{\pi x^2}{2} = \frac{\pi}{2} + n\pi \qquad x^2 = 1 + 2n \qquad x = \sqrt{1 + 2n}$, n integer

Points of inflection at $x = 1, \sqrt{3}, \sqrt{5}$, and $\sqrt{7}$.

5. (a)



(b)

x	0	1	2	3	4	5	6	7	8
$F(x)$	0	$-\frac{1}{2}$	-2	$-\frac{7}{2}$	-4	$-\frac{7}{2}$	-2	$\frac{1}{4}$	3

(c) $f(x) = \begin{cases} -x, & 0 \quad x < 2 \\ x - 4, & 2 \quad x < 6 \\ \frac{1}{2}x - 1, & 6 \quad x \quad 8 \end{cases}$

$$F(x) = \int_0^x f(t)\,dt = \begin{cases} (-x^2/2), & 0 \quad x < 2 \\ (x^2/2) - 4x + 4, & 2 \quad x < 6 \\ (1/4)x^2 - x - 5, & 6 \quad x \quad 8 \end{cases}$$

$F'(x) = f(x)$. F is decreasing on $(0, 4)$ and increasing on $(4, 8)$. Therefore, the minimum is -4 at $x = 4$, and the maximum is 3 at $x = 8$.

(d) $F''(x) = f'(x) = \begin{cases} -1, & 0 < x < 2 \\ 1, & 2 < x < 6 \\ \frac{1}{2}, & 6 < x < 8 \end{cases}$

$x = 2$ is a point of inflection, whereas $x = 6$ is not. (f is not continuous at $x = 6$.)

7. (a) $\displaystyle\int_{-1}^{1} \cos x\,dx \approx \cos\left(-\frac{1}{\sqrt{3}}\right) + \cos\left(\frac{1}{\sqrt{3}}\right) = 2\cos\left(\frac{1}{\sqrt{3}}\right) \approx 1.6758$

$$\int_{-1}^{1} \cos x\,dx = \sin x\Big]_{-1}^{1} = 2\sin(1) \approx 1.6829$$

Error. $|1.6829 - 1.6758| = 0.0071$

(b) $\displaystyle\int_{-1}^{1} \frac{1}{1 + x^2}\,dx \approx \frac{1}{1 + (1/3)} + \frac{1}{1 + (1/3)} = \frac{3}{2}$

(Note: exact answer is $\pi/2 \approx 1.5708$)

(c) Let $p(x) = ax^3 + bx^2 + cx + d$.

$$\int_{-1}^{1} p(x)\,dx = \left[\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx\right]_{-1}^{1} = \frac{2b}{3} + 2d$$

$$p\left(-\frac{1}{\sqrt{3}}\right) + p\left(\frac{1}{\sqrt{3}}\right) = \left(\frac{b}{3} + d\right) + \left(\frac{b}{3} + d\right) = \frac{2b}{3} + 2d$$

9. Consider $F(x) = [f(x)]^2 \qquad F'(x) = 2f(x)\,f'(x)$. Thus,

$$\begin{aligned} \int_a^b f(x)\,f'(x)\,dx &= \int_a^b \frac{1}{2}F'(x)\,dx \\ &= \left[\frac{1}{2}F(x)\right]_a^b \\ &= \frac{1}{2}[F(b) - F(a)] \\ &= \frac{1}{2}[f(b)^2 - f(a)^2] \end{aligned}$$

11. Consider $\displaystyle\int_0^1 x^5\,dx = \frac{x^6}{6}\Big]_0^1 = \frac{1}{6}$.

The corresponding Riemann Sum using right endpoints is

$$\begin{aligned} S(n) &= \frac{1}{n}\left[\left(\frac{1}{n}\right)^5 + \left(\frac{2}{n}\right)^5 + \cdots + \left(\frac{n}{n}\right)^5\right] \\ &= \frac{1}{n^6}[1^5 + 2^5 + \cdots + n^5] \end{aligned}$$

Thus, $\displaystyle\lim_{n \quad \infty} S(n) = \lim_{n \quad \infty} \frac{1^5 + 2^5 + \cdots + n^5}{n^6} = \frac{1}{6}$.

13. By Theorem 4.8, $0 < f(x) \quad M \quad \int_a^b f(x)\,dx \quad \int_a^b M\,dx = M(b-a)$.

Similarly, $m \quad f(x) \quad m(b-a) = \int_a^b m\,dx \quad \int_a^b f(x)\,dx$.

Thus, $m(b-a) \quad \int_a^b f(x)\,dx \quad M(b-a)$. On the interval $[0, 1]$, $1 \quad \sqrt{1+x^4} \quad \sqrt{2}$ and $b - a = 1$.

Thus, $1 \quad \int_0^1 \sqrt{1+x^4}\,dx \quad \sqrt{2}$. $\left(\textbf{Note: } \int_0^1 \sqrt{1+x^4}\,dx \approx 1.0894\right)$

15. Since $-|f(x)| \quad f(x) \quad |f(x)|$,

$$-\int_a^b |f(x)|\,dx \quad \int_a^b f(x)\,dx \quad \int_a^b |f(x)|\,dx \qquad \left|\int_a^b f(x)\,dx\right| \quad \int_a^b |f(x)|\,dx.$$

17. $\displaystyle \frac{1}{365}\int_0^{365} 100{,}000\left[1 + \sin\frac{2\pi(t-60)}{365}\right]dt = \frac{100{,}000}{365}\left[t - \frac{365}{2\pi}\cos\frac{2\pi(t-60)}{365}\right]_0^{365} = 100{,}000$ lbs.