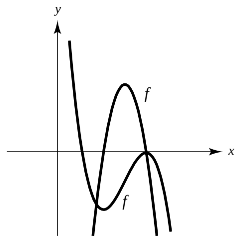
Review Exercises for Chapter 4

2.



4. $u = 3x$

$du = 3\,dx$

$$\int \frac{2}{\sqrt[3]{3x}}\,dx = \frac{2}{3}\int (3x)^{-1/3}(3)\,dx = (3x)^{2/3} + C$$

6. $$\int \frac{x^3 - 2x^2 + 1}{x^2}\,dx = \int (x - 2 + x^{-2})\,dx$$

$$= \frac{1}{2}x^2 - 2x - \frac{1}{x} + C$$

8. $$\int (5\cos x - 2\sec^2 x)\,dx = 5\sin x - 2\tan x + C$$

10. $f''(x) = 6(x - 1)$

$$f'(x) = \int 6(x - 1)\,dx = 3(x - 1)^2 + C_1$$

Since the slope of the tangent line at $(2, 1)$ is 3, it follows that $f'(2) = 3 + C_1 = 3$ when $C_1 = 0$.

$$f'(x) = 3(x - 1)^2$$

$$f(x) = \int 3(x - 1)^2\,dx = (x - 1)^3 + C_2$$

$f(2) = 1 + C_2 = 1$ when $C_2 = 0$.

$$f(x) = (x - 1)^3$$

12. 45 mph $= 66$ ft/sec

30 mph $= 44$ ft/sec

$a(t) = -a$

$v(t) = -at + 66$ since $v(0) = 66$ ft/sec.

$s(t) = -\frac{a}{2}t^2 + 66t$ since $s(0) = 0$.

Solving the system

$$v(t) = -at + 66 = 44$$

$$s(t) = -\frac{a}{2}t^2 + 66t = 264$$

we obtain $t = 24/5$ and $a = 55/12$. We now solve $-(55/12)t + 66 = 0$ and get $t = 72/5$. Thus,

$$s\left(\frac{72}{5}\right) = -\frac{55/12}{2}\left(\frac{72}{5}\right)^2 + 66\left(\frac{72}{5}\right) \approx 475.2 \text{ ft.}$$

Stopping distance from 30 mph to rest is

$$475.2 - 264 = 211.2 \text{ ft.}$$

14. $a(t) = -9.8$ m/sec^2

$v(t) = -9.8t + v_0 = -9.8t + 40$

$s(t) = -4.9t^2 + 40t \quad (s(0) = 0)$

(a) $v(t) = -9.8t + 40 = 0$ when $t = \frac{40}{9.8} \approx 4.08$ sec.

(b) $s(4.08) \approx 81.63$ m

(c) $v(t) = -9.8t + 40 = 20$ when $t = \frac{20}{9.8} \approx 2.04$ sec.

(d) $s(2.04) \approx 61.2$ m

16. $x_1 = 2, x_2 = -1, x_3 = 5, x_4 = 3, x_5 = 7$

(a) $\frac{1}{5}\sum_{i=1}^{5} x_i = \frac{1}{5}(2 - 1 + 5 + 3 + 7) = \frac{16}{5}$

(b) $\sum_{i=1}^{5} \frac{1}{x_i} = \frac{1}{2} - 1 + \frac{1}{5} + \frac{1}{3} + \frac{1}{7} = \frac{37}{210}$

(c) $\sum_{i=1}^{5} (2x_i - x_i^2) = [2(2) - (2)^2] + [2(-1) - (-1)^2] + [2(5) - (5)^2] + [2(3) - (3)^2] + [2(7) - (7)^2] = -56$

(d) $\sum_{i=2}^{5} (x_i - x_{i-1}) = (-1 - 2) + [5 - (-1)] + (3 - 5) + (7 - 3) = 5$

18. $y = 9 - \frac{1}{4}x^2, \Delta x = 1, n = 4$

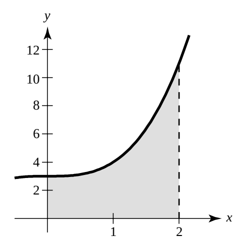
$$S(4) = 1\left[\left(9 - \frac{1}{4}(4)\right) + \left(9 - \frac{1}{4}(9)\right) + \left(9 - \frac{1}{4}(16)\right) + 9 - \frac{1}{4}(25)\right]$$

$$\approx 22.5$$

$$s(4) = 1\left[\left(9 - \frac{1}{4}(9)\right) + \left(9 - \frac{1}{4}(16)\right) + \left(9 - \frac{1}{4}(25)\right) + (9 - 9)\right]$$

$$\approx 14.5$$

20. $y = x^2 + 3, \Delta x = \frac{2}{n}$ right endpoints



$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(ci)\,\Delta x$$

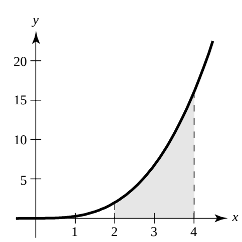
$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^2 + 3\right]\left(\frac{2}{n}\right)$$

$$= \lim_{n \to \infty} \frac{2}{n} \sum_{i=1}^{n} \left[\frac{4i^2}{n^2} + 3\right]$$

$$= \lim_{n \to \infty} \frac{2}{n}\left[\frac{4}{n^2}\frac{n(n + 1)(2n + 1)}{6} + 3n\right]$$

$$= \lim_{n \to \infty} \left[\frac{4}{3}\frac{(n + 1)(2n + 1)}{n^2} + 6\right] = \frac{8}{3} + 6 = \frac{26}{3}$$

22. $y = \frac{1}{4}x^3, \Delta x = \frac{2}{n}$



$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(ci)\,\Delta x$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{4}\left(2 + \frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$$

$$= \lim_{n \to \infty} \frac{1}{2n} \sum_{i=1}^{n}\left[8 + \frac{24i}{n} + \frac{24i^2}{n^2} + \frac{8i^3}{n^3}\right]$$

$$= \lim_{n \to \infty} \frac{4}{n} \sum_{i=1}^{n} \left[1 + \frac{3i}{n} + \frac{3i^2}{n^2} + \frac{i^3}{n^3}\right]$$

$$= \lim_{n \to \infty} \frac{4}{n}\left[n + \frac{3}{n}\frac{n(n + 1)}{2} + \frac{3}{n^2}\frac{n(n + 1)(2n + 1)}{6} + \frac{1}{n^3}\frac{n^2(n + 1)^2}{4}\right]$$

$$= 4 + 6 + 4 + 1 = 15$$

24. (a) $S = m\left(\frac{b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{2b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{3b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{4b}{4}\right)\left(\frac{b}{4}\right) = \frac{mb^2}{16}(1 + 2 + 3 + 4) = \frac{5mb^2}{8}$

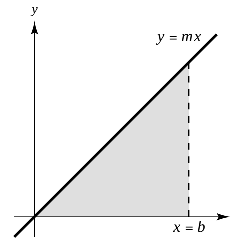
$s = m(0)\left(\frac{b}{4}\right) + m\left(\frac{b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{2b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{3b}{4}\right)\left(\frac{b}{4}\right) = \frac{mb^2}{16}(1 + 2 + 3) = \frac{3mb^2}{8}$

(b) $S(n) = \sum_{i=1}^{n} f\left(\frac{bi}{n}\right)\left(\frac{b}{n}\right) = \sum_{i=1}^{n}\left(\frac{mbi}{n}\right)\left(\frac{b}{n}\right) = m\left(\frac{b}{n}\right)^2\sum_{i=1}^{n} i = \frac{mb^2}{n^2}\left(\frac{n(n+1)}{2}\right) = \frac{mb^2(n+1)}{2n}$

$s(n) = \sum_{i=0}^{n-1} f\left(\frac{bi}{n}\right)\left(\frac{b}{n}\right) = \sum_{i=0}^{n-1} m\left(\frac{bi}{n}\right)\left(\frac{b}{n}\right) = m\left(\frac{b}{n}\right)^2\sum_{i=0}^{n-1} i = \frac{mb^2}{n^2}\left(\frac{(n-1)n}{2}\right) = \frac{mb^2(n-1)}{2n}$

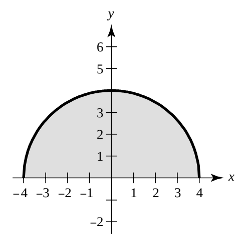
(c) $\text{Area} = \lim_{n\to\infty}\frac{mb^2(n+1)}{2n} = \lim_{n\to\infty}\frac{mb^2(n-1)}{2n} = \frac{1}{2}mb^2 = \frac{1}{2}(b)(mb) = \frac{1}{2}(\text{base})(\text{height})$

(d) $\int_0^b mx\,dx = \left[\frac{1}{2}mx^2\right]_0^b = \frac{1}{2}mb^2$



26. $\lim_{\|\Delta\|\to\infty}\sum_{i=1}^{n} 3c_i(9 - c_i^2)\,\Delta x_i = \int_1^3 3x(9 - x^2)\,dx$

28.



$\int_{-4}^{4}\sqrt{16 - x^2}\,dx = \frac{1}{2}\pi(4)^2 = 8\pi$ (semicircle)

30. (a) $\int_0^6 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx = 4 + (-1) = 3$

(b) $\int_6^3 f(x)\,dx = -\int_3^6 f(x)\,dx = -(-1) = 1$

(c) $\int_4^4 f(x)\,dx = 0$

(d) $\int_3^6 -10 f(x)\,dx = -10\int_3^6 f(x)\,dx = -10(-1) = 10$

32. $\int_1^3 \frac{12}{x^3}\,dx = \left[\frac{12x^{-2}}{-2}\right]_1^3 = \left[\frac{-6}{x^2}\right]_1^3 = \frac{-6}{9} + 6 = \frac{16}{3}$, (d)

34. $\int_{-1}^{1}(t^2 + 2)\,dt = \left[\frac{t^3}{3} + 2t\right]_{-1}^{1} = \frac{14}{3}$

36. $\int_{-2}^{2}(x^4 + 2x^2 - 5)\,dx = \left[\frac{x^5}{5} + \frac{2x^3}{3} - 5x\right]_{-2}^{2}$

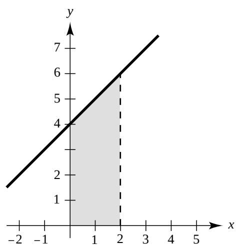
$= \left(\frac{32}{5} + \frac{16}{3} - 10\right) - \left(-\frac{32}{5} - \frac{16}{3} + 10\right)$

$= \frac{52}{15}$

38. $\int_1^2\left(\frac{1}{x^2} - \frac{1}{x^3}\right)dx = \int_1^2 (x^{-2} - x^{-3})\,dx = \left[-\frac{1}{x} + \frac{1}{2x^2}\right]_1^2 = \left(-\frac{1}{2} + \frac{1}{8}\right) - \left(-1 + \frac{1}{2}\right) = \frac{1}{8}$

40. $\int_{-\pi/4}^{\pi/4}\sec^2 t\,dt = \left[\tan t\right]_{-\pi/4}^{\pi/4} = 1 - (-1) = 2$

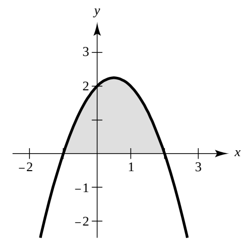
42. $\int_0^2 (x + 4)\,dx = \left[\frac{x^2}{2} + 4x\right]_0^2 = 10$



44. $\int_{-1}^2 (-x^2 + x + 2)\,dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^2$

$$= \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

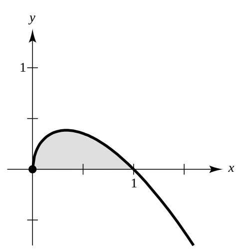
$$= \frac{10}{3} + \frac{7}{6} = \frac{9}{2}$$



46. $\int_0^1 \sqrt{x}(1 - x)\,dx = (x^{1/2} - x^{3/2})\,dx$

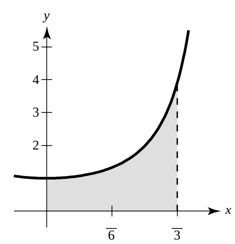
$$= \left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^1$$

$$= \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$$



48. Area $= \int_0^{\pi/3} \sec^2 x\,dx$

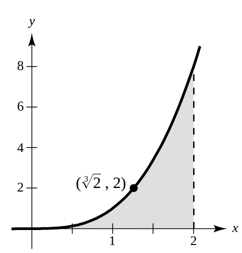
$$= \tan x\Big]_0^{\pi/3} = \sqrt{3}$$



50. $\frac{1}{2 - 0}\int_0^2 x^3\,dx = \left[\frac{x^4}{8}\right]_0^2 = 2$

$x^3 = 2$

$x = \sqrt[3]{2}$



52. $F'(x) = \frac{1}{x^2}$

54. $F'(x) = \csc^2 x$

56. $\int \left(x + \frac{1}{x}\right)^2 dx = \int (x^2 + 2 + x^{-2})\,dx$

$$= \frac{x^3}{3} + 2x - \frac{1}{x} + C$$

58. $u = x^3 + 3, du = 3x^2\,dx$

$$\int x^2\sqrt{x^3+3}\,dx = \frac{1}{3}\int (x^3+3)^{1/2}\,3x^2\,dx = \frac{2}{9}(x^3+3)^{3/2} + C$$

60. $u = x^2 + 6x - 5, du = (2x+6)\,dx$

$$\int \frac{x+3}{(x^2+6x-5)^2} = \frac{1}{2}\int \frac{2x+6}{(x^2+6x-5)^2}\,dx = \frac{-1}{2}(x^2+6x-5)^{-1} + C = \frac{-1}{2(x^2+6x-5)} + C$$

62. $\int x \sin 3x^2\,dx = \frac{1}{6}\int (\sin 3x^2)(6x)\,dx = -\frac{1}{6}\cos 3x^2 + C$

64. $\int \frac{\cos x}{\sqrt{\sin x}}\,dx = \int (\sin x)^{-1/2}\cos x\,dx = 2(\sin x)^{1/2} + C = 2\sqrt{\sin x} + C$

66. $\int \sec 2x \tan 2x\,dx = \frac{1}{2}\int (\sec 2x \tan 2x)(2)\,dx = \frac{1}{2}\sec 2x + C$

68. $\int \cot^4 \alpha \csc^2 \alpha\,d\alpha = -\int (\cot \alpha)^4(-\csc^2 \alpha)\,d\alpha = -\frac{1}{5}\cot^5 \alpha + C$

70. $\int_0^1 x^2(x^3+1)^3\,dx = \frac{1}{3}\int_0^1 (x^3+1)^3(3x^2)\,dx = \frac{1}{12}\Big[(x^3+1)^4\Big]_0^1 = \frac{1}{12}(16-1) = \frac{5}{4}$

72. $\int_3^6 \frac{x}{3\sqrt{x^2-8}}\,dx = \frac{1}{6}\int_3^6 (x^2-8)^{-1/2}(2x)\,dx = \left[\frac{1}{3}(x^2-8)^{1/2}\right]_3^6 = \frac{1}{3}\left(2\sqrt{7}-1\right)$

74. $u = x + 1, x = u - 1, dx = du$

When $x = -1, u = 0$. When $x = 0, u = 1$.

$$2\pi\int_{-1}^{0} x^2\sqrt{x+1}\,dx = 2\pi\int_0^1 (u-1)^2\sqrt{u}\,du$$

$$= 2\pi\int_0^1 (u^{5/2} - 2u^{3/2} + u^{1/2})\,du = 2\pi\left[\frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_0^1 = \frac{32\pi}{105}$$

76. $\int_{-\pi/4}^{\pi/4} \sin 2x\,dx = 0$ since $\sin 2x$ is an odd function.

78. $u = 1 - x, x = 1 - u, dx = -du$

When $x = a, u = 1 - a$. When $x = b, u = 1 - b$.

$$P_{a,b} = \int_a^b \frac{1155}{32}x^3(1-x)^{3/2}\,dx = \frac{1155}{32}\int_{1-a}^{1-b} -(1-u)^3 u^{3/2}\,du$$

$$= \frac{1155}{32}\int_{1-a}^{1-b} (u^{9/2} - 3u^{7/2} + 3u^{5/2} - u^{3/2})\,du = \frac{1155}{32}\left[\frac{2}{11}u^{11/2} - \frac{2}{3}u^{9/2} + \frac{6}{7}u^{7/2} - \frac{2}{5}u^{5/2}\right]_{1-a}^{1-b}$$

$$= \frac{1155}{32}\left[\frac{2u^{5/2}}{1155}(105u^3 - 385u^2 + 495u - 231)\right]_{1-a}^{1-b} = \left[\frac{u^{5/2}}{16}(105u^3 - 385u^2 + 495u - 231)\right]_{1-a}^{1-b}$$

(a) $P_{0,\,0.25} = \left[\frac{u^{5/2}}{16}(105u^3 - 385u^2 + 495u - 231)\right]_1^{0.75} \approx 0.025 = 2.5\%$

(b) $P_{0.5,\,1} = \left[\frac{u^{5/2}}{16}(105u^3 - 385u^2 + 495u - 231)\right]_{0.5}^{0} \approx 0.736 = 73.6\%$

80. $\int_0^2 1.75 \sin \frac{\pi t}{2}\, dt = -\frac{2}{\pi}\left[1.75 \cos \frac{\pi t}{2}\right]_0^2 = -\frac{2}{\pi}(1.75)(-1 - 1) = \frac{7}{\pi} \approx 2.2282$ liters

Increase is

$$\frac{7}{\pi} - \frac{5.1}{\pi} = \frac{1.9}{\pi} \approx 0.6048 \text{ liters.}$$

82. Trapezoidal Rule $(n = 4)$: $\int_0^1 \frac{x^{3/2}}{3 - x^2}\, dx \approx \frac{1}{8}\left[0 + \frac{2(1/4)^{3/2}}{3 - (1/4)^2} + \frac{2(1/2)^{3/2}}{3 - (1/2)^2} + \frac{2(3/4)^{3/2}}{3 - (3/4)^2} + \frac{1}{2}\right] \approx 0.172$

Simpson's Rule $(n = 4)$: $\int_0^1 \frac{x^{3/2}}{3 - x^2}\, dx \approx \frac{1}{12}\left[0 + \frac{4(1/4)^{3/2}}{3 - (1/4)^2} + \frac{2(1/2)^{3/2}}{3 - (1/2)^2} + \frac{4(3/4)^{3/2}}{3 - (3/4)^2} + \frac{1}{2}\right] \approx 0.166$

Graphing utility: 0.166

84. Trapezoidal Rule $(n = 4)$: $\int_0^{\pi} \sqrt{1 + \sin^2 x}\, dx \approx 3.820$

Simpson's Rule $(n = 4)$: 3.820

Graphing utility: 3.820

Problem Solving for Chapter 4

2. (a) $F(x) = \int_2^x \sin t^2\, dt$

x	0	1.0	1.5	1.9	2.0	2.1	2.5	3.0	4.0	5.0
$F(x)$	-0.8048	-0.4945	-0.0265	0.0611	0	-0.0867	-0.3743	-0.0312	-0.0576	-0.2769

(b) $G(x) = \frac{1}{x - 2}\int_2^x \sin t^2\, dt$

x	1.9	1.95	1.99	2.01	2.05	2.1
$G(x)$	-0.6106	-0.6873	-0.7436	-0.7697	-0.8174	-0.8671

$\lim_{x \to 2} G(x) \approx -0.75$

(c) $F'(2) = \lim_{x \to 2} \frac{F(x) - F(2)}{x - 2}$

$= \lim_{x \to 2} \frac{1}{x - 2}\int_2^x \sin t^2\, dt$

$= \lim_{x \to 2} G(x)$

Since $F'(x) = \sin x^2$, $F'(2) = \sin 4 = \lim_{x \to 2} G(x)$.

(**Note:** $\sin 4 \approx -0.7568$)

4. Let d be the distance traversed and a be the uniform acceleration.
We can assume that $v(0) = 0$ and $s(0) = 0$. Then

$$a(t) = a$$

$$v(t) = at$$

$$s(t) = \frac{1}{2}at^2.$$

$s(t) = d$ when $t = \sqrt{\frac{2d}{a}}$.

The highest speed is $v = a\sqrt{\frac{2d}{a}} = \sqrt{2ad}$.

The lowest speed is $v = 0$.

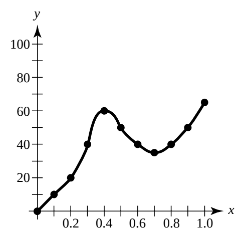
The mean speed is $\frac{1}{2}\left(\sqrt{2ad} + 0\right) = \sqrt{\frac{ad}{2}}$.

The time necessary to traverse the distance d at the mean speed is

$$t = \frac{d}{\sqrt{ad/2}} = \sqrt{\frac{2d}{a}}$$

which is the same as the time calculated above.

6. (a)



(b) v is increasing (positive acceleration) on $(0, 0.4)$ and $(0.7, 1.0)$.

(c) Average acceleration $= \dfrac{v(0.4) - v(0)}{0.4 - 0} = \dfrac{60 - 0}{0.4} = 150$ mi/hr^2

(d) This integral is the total distance traveled in miles.

$$\int_0^1 v(t)\,dt \approx \frac{1}{10}[0 + 2(20) + 2(60) + 2(40) + 2(40) + 65] = \frac{385}{10} = 38.5 \text{ miles}$$

(e) One approximation is

$$a(0.8) \approx \frac{v(0.9) - v(0.8)}{0.9 - 0.8} = \frac{50 - 40}{0.1} = 100 \text{ mi/hr}^2$$

(other answers possible)

8. $\int_0^x f(t)(x-t)\,dt = \int_0^x xf(t)\,dt - \int_0^x tf(t)\,dt = x\int_0^x f(t)\,dt - \int_0^x tf(t)\,dt$

Thus, $\frac{d}{dx}\int_0^x f(t)(x-t)\,dt = xf(x) + \int_0^x f(t)\,dt - xf(x) = \int_0^x f(t)\,dt$

Differentiating the other integral,

$$\frac{d}{dx}\int_0^x \left(\int_0^x f(v)\,dv\right)dt = \int_0^x f(v)\,dv.$$

Thus, the two original integrals have equal derivatives,

$$\int_0^x f(t)(x-t)\,dt = \int_0^x \left(\int_0^t f(v)\,dv\right)dt + C$$

Letting $x = 0$, we see that $C = 0$.

10. Consider $\int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big]_0^1 = \frac{2}{3}$. The corresponding

Riemann Sum using right-hand endpoints is

$$S(n) = \frac{1}{n}\left[\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right]$$

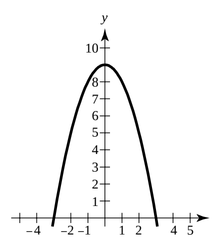
$$= \frac{1}{n^{3/2}}\left[\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n}\right]$$

Thus, $\lim_{n \to \infty} \frac{\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n}}{n^{3/2}} = \frac{2}{3}$.

12. (a) Area $= \int_{-3}^3 (9 - x^2)\,dx = 2\int_0^3 (9 - x^2)\,dx$

$$= 2\left[9x - \frac{x^3}{3}\right]_0^3$$

$$= 2[27 - 9] = 36$$



(b) Base $= 6$, height $= 9$. Area $= \frac{2}{3}bh = \frac{2}{3}(6)(9) = 36$.

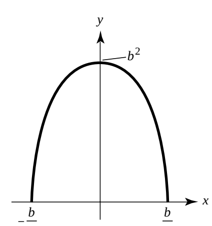
(c) Let the parabola be given by $y = b^2 - a^2x^2,\ a, b > 0$.

Area $= 2\int_0^{b/a} (b^2 - a^2x^2)\,dx$

$$= 2\left[b^2x - a^2\frac{x^3}{3}\right]_0^{b/a}$$

$$= 2\left[b^2\left(\frac{b}{a}\right) - \frac{a^2}{3}\left(\frac{b}{a}\right)^3\right]$$

$$= 2\left[\frac{b^3}{a} - \frac{1}{3}\frac{b^3}{a}\right] = \frac{4}{3}\frac{b^3}{a}$$



Base $= \frac{2b}{a}$, height $= b^2$

Archimedes' Formula: Area $= \frac{2}{3}\left(\frac{2b}{a}\right)(b^2) = \frac{4}{3}\frac{b^3}{a}$

14. (a) $(1 + i)^3 = 1 + 3i + 3i^2 + i^3 \qquad (1 + i)^3 - i^3 = 3i^2 + 3i + 1$

(b) $3i^2 + 3i + 1 = (i + 1)^3 - i^3$

$$\sum_{i=1}^{n}(3i^2 + 3i + 1) = \sum_{i=1}^{n}[(i + 1)^3 - i^3]$$
$$= (2^3 - 1^3) + (3^3 - 2^3) + \cdots + [((n + 1)^3 - n^3)]$$
$$= (n + 1)^3 - 1$$

Hence, $(n + 1)^3 = \sum_{i=1}^{n}(3i^2 + 3i + 1) + 1$

(c) $(n + 1)^3 - 1 = \sum_{i=1}^{n}(3i^2 + 3i + 1) = \sum_{i=1}^{n}3i^2 + \dfrac{3(n)(n + 1)}{2} + n$

$$\sum_{i=1}^{n}3i^2 = n^3 + 3n^2 + 3n - \frac{3n(n + 1)}{2} - n$$
$$= \frac{2n^3 + 6n^2 + 6n - 3n^2 - 3n - 2n}{2}$$
$$= \frac{2n^3 + 3n^2 + n}{2}$$
$$= \frac{n(n + 1)(2n + 1)}{2}$$
$$\sum_{i=1}^{n}i^2 = \frac{n(n + 1)(2n + 1)}{6}$$

16. (a) $C = 0.1\int_{8}^{20}\left[12 \sin \dfrac{\pi(t - 8)}{12}\right] dt = \left[-\dfrac{14.4}{\pi} \cos \dfrac{\pi(t - 8)}{12}\right]_{8}^{20} = \dfrac{-14.4}{\pi}(-1 - 1) \approx \$9.17$

(b) $C = 0.1\int_{10}^{18}\left[12 \sin \dfrac{\pi(t - 8)}{12} - 6\right] dt = \left[-\dfrac{14.4}{\pi} \cos \dfrac{\pi(t - 8)}{12} - 0.6t\right]_{10}^{18}$

$$= \left[\frac{-14.4}{\pi}\left(-\frac{\sqrt{3}}{2}\right) - 10.8\right] - \left[\frac{-14.4}{\pi}\left(\frac{\sqrt{3}}{2}\right) - 6\right] \approx \$3.14$$

Savings $\approx 9.17 - 3.14 = \$6.03$.

18. (a) Let $A = \displaystyle\int_0^b \frac{f(x)}{f(x) + f(b - x)}\, dx$.

Let $u = b - x$, $du = -dx$.

$$A = \int_b^0 \frac{f(b - u)}{f(b - u) + f(u)}(-du)$$
$$= \int_0^b \frac{f(b - u)}{f(b - u) + f(u)}\, du$$
$$= \int_0^b \frac{f(b - x)}{f(b - x) + f(x)}\, dx$$

Then,

$$2A = \int_0^b \frac{f(x)}{f(x) + f(b - x)}\, dx + \int_0^b \frac{f(b - x)}{f(b - x) + f(x)}\, dx$$
$$= \int_0^b 1\, dx = b.$$

Thus, $A = \dfrac{b}{2}$.

(b) $b = 1 \qquad \displaystyle\int_0^1 \frac{\sin x}{\sin(1 - x) + \sin x}\, dx = \frac{1}{2}$