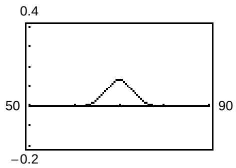
84. (a) $f(x) = \dfrac{1}{3\sqrt{2\pi}}e^{-(x-70)^2/18}$

$\displaystyle\int_{50}^{90} f(x)\,dx \approx 1.0$



(b) $P(72 \le x < \infty) \approx 0.2525$

(c) $0.5 - P(70 \le x \le 72) \approx 0.5 - 0.2475 = 0.2525$

These are the same answers because by symmetry,

$P(70 \le x < \infty) = 0.5$

and

$$0.5 = P(70 \le x < \infty)$$
$$= P(70 \le x \le 72) + P(72 \le x < \infty).$$

86. False. This is equivalent to Exercise 85.

88. True

Review Exercises for Chapter 7

2. $\displaystyle\int xe^{x^2-1}\,dx = \frac{1}{2}\int e^{x^2-1}(2x)\,dx$

$$= \frac{1}{2}e^{x^2-1} + C$$

4. $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int (1-x^2)^{-1/2}(-2x)\,dx$

$$= -\frac{1}{2}\frac{(1-x^2)^{1/2}}{1/2} + C$$
$$= -\sqrt{1-x^2} + C$$

6. $\displaystyle\int 2x\sqrt{2x-3}\,dx = \int (u^4 + 3u^2)\,du = \frac{u^5}{5} + u^3 + C$

$$= \frac{2(2x-3)^{3/2}}{5}(x+1) + C$$

$u = \sqrt{2x-3},\ x = \dfrac{u^2+3}{2},\ dx = u\,du$

8. $\dfrac{x^4 + 2x^2 + x + 1}{x^4 + 2x^2 + 1} = 1 + \dfrac{x}{(x^2+1)^2}$

$$\int \frac{x^4 + 2x^2 + x + 1}{(x^2+1)^2}\,dx = \int dx + \frac{1}{2}\int \frac{2x}{(x^2+1)^2}\,dx$$
$$= x - \frac{1}{2(x^2+1)} + C$$

10. $\displaystyle\int (x^2-1)e^x\,dx = (x^2-1)e^x - 2\int xe^x\,dx = (x^2-1)e^x - 2xe^x + 2\int e^x\,dx = e^x(x^2 - 2x + 1) + 1$

(1) $dv = e^x\,dx \implies v = e^x$

$u = x^2 - 1 \implies du = 2x\,dx$

(2) $dv = e^x\,dx \implies v = e^x$

$u = x \implies du = dx$

12. $u = \arctan 2x,\ du = \dfrac{2}{1+4x^2}\,dx,\ dv = dx, v = x$

$$\int \arctan 2x\,dx = x\arctan 2x - \int \frac{2x}{1+4x^2}\,dx$$
$$= x\arctan 2x - \frac{1}{4}\ln(1+4x^2) + C$$

14. $\displaystyle\int \ln\sqrt{x^2-1}\,dx = \frac{1}{2}\int \ln(x^2-1)\,dx$

$$= \frac{1}{2}x\ln|x^2-1| - \int \frac{x^2}{x^2-1}\,dx$$
$$= \frac{1}{2}x\ln|x^2-1| - \int dx - \int \frac{1}{x^2-1}\,dx$$
$$= \frac{1}{2}x\ln|x^2-1| - x - \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$$

$dv = dx \implies v = x$

$u = \ln(x^2-1) \implies du = \dfrac{2x}{x^2-1}\,dx$

16. $\displaystyle\int e^x \arctan(e^x)\,dx = e^x \arctan(e^x) - \int \frac{e^{2x}}{1 + e^{2x}}\,dx$

$\displaystyle = e^x \arctan(e^x) - \frac{1}{2}\ln(1 + e^{2x}) + C$

$dv = e^x\,dx \implies v = e^x$

$\displaystyle u = \arctan e^x \implies du = \frac{e^x}{1 + e^{2x}}\,dx$

18. $\displaystyle\int \sin^2 \frac{\pi x}{2}\,dx = \int \frac{1}{2}(1 - \cos \pi x)\,dx = \frac{1}{2}\left[x - \frac{1}{\pi}\sin \pi x\right] + C = \frac{1}{2\pi}[\pi x - \sin \pi x] + C$

20. $\displaystyle\int \tan\theta \sec^4\theta\,d\theta = \int (\tan^3\theta + \tan\theta)\sec^2\theta\,d\theta = \frac{1}{4}\tan^4\theta + \frac{1}{2}\tan^2\theta + C_1$

or

$\displaystyle\int \tan\theta \sec^4\theta\,d\theta = \int \sec^3\theta(\sec\theta\tan\theta)\,d\theta = \frac{1}{4}\sec^4\theta + C_2$

22. $\displaystyle\int \cos 2\theta(\sin\theta + \cos\theta)^2\,d\theta = \int (\cos^2\theta - \sin^2\theta)(\sin\theta + \cos\theta)^2\,d\theta$

$\displaystyle = \int (\sin\theta + \cos\theta)^3(\cos\theta - \sin\theta)\,d\theta = \frac{1}{4}(\sin\theta + \cos\theta)^4 + C$

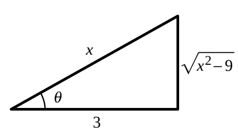
24. $\displaystyle\int \frac{\sqrt{x^2 - 9}}{x}\,dx = \int \frac{3\tan\theta}{3\sec\theta}(3\sec\theta\tan\theta\,d\theta)$

$\displaystyle = 3\int \tan^2\theta\,d\theta$

$\displaystyle = 3\int (\sec^2\theta - 1)\,d\theta$

$= 3(\tan\theta - \theta) + C$

$\displaystyle = \sqrt{x^2 - 9} - 3\,\mathrm{arcsec}\left(\frac{x}{3}\right) + C$



$x = 3\sec\theta,\ dx = 3\sec\theta\tan\theta\,d\theta,\ \sqrt{x^2 - 9} = 3\tan\theta$

26. $\displaystyle\int \sqrt{9 - 4x^2}\,dx = \frac{1}{2}\int \sqrt{9 - (2x)^2}\,(2)\,dx$

$\displaystyle = \frac{1}{2}\cdot\frac{1}{2}\left[9\arcsin\frac{2x}{3} + 2x\sqrt{9 - 4x^2}\right] + C$

$\displaystyle = \frac{9}{4}\arcsin\frac{2x}{3} + \frac{x}{2}\sqrt{9 - 4x^2} + C$

28. $\displaystyle\int \frac{\sin\theta}{1 + 2\cos^2\theta}\,d\theta = \frac{-1}{\sqrt{2}}\int \frac{1}{1 + 2\cos^2\theta}\left(-\sqrt{2}\sin\theta\right)d\theta$

$\displaystyle = \frac{-1}{\sqrt{2}}\arctan\left(\sqrt{2}\cos\theta\right) + C$

$u = \sqrt{2}\cos\theta,\ du = -\sqrt{2}\sin\theta\,d\theta$

30. (a) $\displaystyle\int x\sqrt{4+x}\,dx = 64\int \tan^3\theta\sec^3\theta\,d\theta$

$$= 64\int(\sec^4\theta - \sec^2\theta)\sec\theta\tan\theta\,d\theta$$

$$= \frac{64\sec^3\theta}{15}(3\sec^3\theta - 5) + C$$

$$= \frac{2(4+x)^{3/2}}{15}(3x-8) + C$$

$x = 4\tan^2\theta,\ dx = 8\tan\theta\sec^2\theta\,d\theta,$

$\sqrt{4+x} = 2\sec\theta$

(b) $\displaystyle\int x\sqrt{4+x}\,dx = 2\int(u^4 - 4u^2)\,du$

$$= \frac{2u^3}{15}(3u^2 - 20) + C$$

$$= \frac{2(4+x)^{3/2}}{15}(3x-8) + C$$

$u^2 = 4+x,\ dx = 2u\,du$

(c) $\displaystyle\int x\sqrt{4+x}\,dx = \int(u^{3/2} - 4u^{1/2})\,du$

$$= \frac{2u^{3/2}}{15}(3u-20) + C$$

$$= \frac{2(4+x)^{3/2}}{15}(3x-8) + C$$

$u = 4+x,\ du = dx$

(d) $\displaystyle\int x\sqrt{4+x}\,dx = \frac{2x}{3}(4+x)^{3/2} - \frac{2}{3}\int(4+x)^{3/2}\,dx$

$$= \frac{2x}{3}(4+x)^{3/2} - \frac{4}{15}(4+x)^{5/2} + C$$

$$= \frac{2(4+x)^{3/2}}{15}(3x-8) + C$$

$dv = \sqrt{4+x}\,dx \implies v = \dfrac{2}{3}(4+x)^{3/2}$

$u = x \implies du = dx$

32. $\displaystyle\frac{2x^3 - 5x^2 + 4x - 4}{x^2 - x} = 2x - 3 + \frac{4}{x} - \frac{3}{x-1}$

$$\int\frac{2x^3 - 5x^2 + 4x - 4}{x^2 - x}\,dx = \int\left(2x - 3 + \frac{4}{x} - \frac{3}{x-1}\right)dx = x^2 - 3x + 4\ln|x| - 3\ln|x-1| + C$$

34. $\displaystyle\frac{4x-2}{3(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$

$4x - 2 = 3A(x-1) + 3B$

Let $x = 1$: $2 = 3B \implies B = \dfrac{2}{3}$

Let $x = 2$: $6 = 3A + 3B \implies A = \dfrac{4}{3}$

$$\int\frac{4x-2}{3(x-1)^2}\,dx = \frac{4}{3}\int\frac{1}{x-1}\,dx + \frac{2}{3}\int\frac{1}{(x-1)^2}\,dx = \frac{4}{3}\ln|x-1| - \frac{2}{3(x-1)} + C = \frac{2}{3}\left(2\ln|x-1| - \frac{1}{x-1}\right) + C$$

36. $\displaystyle\int\frac{\sec^2\theta}{\tan\theta(\tan\theta - 1)}\,d\theta = \int\frac{1}{u(u-1)}\,du = \int\frac{1}{u-1}\,du - \int\frac{1}{u}\,du$

$$= \ln|u-1| - \ln|u| + C = \ln\left|\frac{\tan\theta - 1}{\tan\theta}\right| + C = \ln|1 - \cot\theta| + C$$

$u = \tan\theta,\ du = \sec^2\theta\,d\theta$

$\displaystyle\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$

$1 = A(u-1) + Bu$

Let $u = 0$: $1 = -A \implies A = -1$

Let $u = 1$: $1 = B$

38. $\displaystyle\int \frac{x}{\sqrt{2+3x}}\,dx = \frac{-2(4-3x)}{27}\sqrt{2+3x} + C$ (Formula 21)

$$= \frac{6x-8}{27}\sqrt{2+3x} + C$$

40. $\displaystyle\int \frac{x}{1+e^{x^2}}\,dx = \frac{1}{2}\int \frac{1}{1+e^u}\,du$ $(u = x^2)$

$$= \frac{1}{2}[u - \ln(1+e^u)] + C \quad \text{(Formula 84)}$$

$$= \frac{1}{2}[x^2 - \ln(1+e^{x^2})] + C$$

42. $\displaystyle\int \frac{3}{2x\sqrt{9x^2-1}}\,dx = \frac{3}{2}\int \frac{1}{3x\sqrt{(3x)^2-1}}\,3\,dx$ $(u = 3x)$

$$= \frac{3}{2}\operatorname{arcsec}|3x| + C \quad \text{(Formula 33)}$$

44. $\displaystyle\int \frac{1}{1+\tan \pi x}\,dx = \frac{1}{\pi}\int \frac{1}{1+\tan \pi x}(\pi)dx$ $(u = \pi x)$

$$= \frac{1}{\pi}\frac{1}{2}\big[\pi x + \ln|\cos \pi x + \sin \pi x|\big] + C \quad \text{(Formula 71)}$$

46. $\displaystyle\int \tan^n x\,dx = \int \tan^{n-2}x(\sec^2 x - 1)\,dx$

$$= \int \tan^{n-2} x \sec^2 x\,dx - \int \tan^{n-2} x\,dx$$

$$= \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$$

48. $\displaystyle\int \frac{\csc\sqrt{2x}}{\sqrt{x}}\,dx = \sqrt{2}\int \csc\sqrt{2x}\left(\frac{1}{\sqrt{2x}}\right)dx$

$$= -\sqrt{2}\ln\left|\csc\sqrt{2x} + \cot\sqrt{2x}\right| + C$$

$u = \sqrt{2x}$, $du = \dfrac{1}{\sqrt{2x}}\,dx$

50. $\displaystyle\int \sqrt{1+\sqrt{x}}\,dx = \int u(4u^3 - 4u)\,du = \int (4u^4 - 4u^2)\,du = \frac{4u^5}{5} - \frac{4u^3}{3} + C = \frac{4}{15}\big(1+\sqrt{x}\big)^{3/2}\big(3\sqrt{x} - 2\big) + C$

$u = \sqrt{1+\sqrt{x}}$, $x = u^4 - 2u^2 + 1$, $dx = (4u^3 - 4u)\,du$

52. $\dfrac{3x^3+4x}{(x^2+1)^2} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2}$

$3x^3 + 4x = (Ax+B)(x^2+1) + Cx + D$

$\quad = Ax^3 + Bx^2 + (A+C)x + (B+D)$

$A = 3, B = 0, A + C = 4 \implies C = 1,$

$B + D = 0 \implies D = 0$

$$\int \frac{3x^3+4x}{(x^2+1)^2}\,dx = 3\int \frac{x}{x^2+1}\,dx + \int \frac{x}{(x^2+1)^2}\,dx$$

$$= \frac{3}{2}\ln(x^2+1) - \frac{1}{2(x^2+1)} + C$$

54. $\displaystyle\int (\sin\theta + \cos\theta)^2\,d\theta = \int (\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta)\,d\theta$

$$= \int (1 + \sin 2\theta)\,d\theta = \theta - \frac{1}{2}\cos 2\theta + C = \frac{1}{2}(2\theta - \cos 2\theta) + C$$

56. $$y = \int \frac{\sqrt{4 - x^2}}{2x}\, dx = \int \frac{2\cos\theta(2\cos\theta)\,d\theta}{4\sin\theta}$$
$$= \int (\csc\theta - \sin\theta)\, d\theta$$
$$= \left[-\ln|\csc\theta + \cos\theta| + \cos\theta\right] + C$$
$$= -\ln\left|\frac{2 + \sqrt{4 - x^2}}{x}\right| + \frac{\sqrt{4 - x^2}}{2} + C$$

$x = 2\sin\theta,\ dx = 2\cos\theta\, d\theta,\ \sqrt{4 - x^2} = 2\cos\theta$

58. $$y = \int \sqrt{1 - \cos\theta}\, d\theta = \int \frac{\sin\theta}{\sqrt{1 + \cos\theta}}\, d\theta = -\int (1 + \cos\theta)^{-1/2}(-\sin\theta)d\theta = -2\sqrt{1 + \cos\theta} + C$$

$u = 1 + \cos\theta,\ du = -\sin\theta\, d\theta$

60. $$\int_0^1 \frac{x}{(x - 2)(x - 4)}\, dx = \Big[2\ln|x - 4| - \ln|x - 2|\Big]_0^1$$
$$= 2\ln 3 - 2\ln 4 + \ln 2$$
$$= \ln\frac{9}{8} \approx 0.118$$

62. $$\int_0^2 xe^{3x}\, dx = \left[\frac{e^{3x}}{9}(3x - 1)\right]_0^2 = \frac{1}{9}(5e^6 + 1) \approx 224.238$$

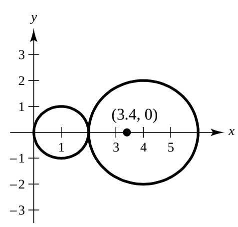
64. $$\int_0^3 \frac{x}{\sqrt{1 + x}}\, dx = \left[\frac{-2(2 - x)}{3}\sqrt{1 + x}\right]_0^3 = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$$

66. $$A = \int_0^4 \frac{1}{25 - x^2}\, dx$$
$$= \left[-\frac{1}{10}\ln\left|\frac{x - 5}{x + 5}\right|\right]_0^4 = -\frac{1}{10}\ln\frac{1}{9} = \frac{1}{10}\ln 9 \approx 0.220$$

68. By symmetry, $\bar{y} = 0$.

$$A = \pi + 4\pi = 5\pi$$
$$\bar{x} = \frac{1(\pi) + 4(4\pi)}{\pi + 4\pi}$$
$$= \frac{17\pi}{5\pi} = 3.4$$

$(\bar{x}, \bar{y}) = (3.4, 0)$



70. $$s = \int_0^{\pi} \sqrt{1 + \sin^2 2x}\, dx \approx 3.82$$

72. $$\lim_{x\to 0} \frac{\sin \pi x}{\sin 2\pi x} = \lim_{x\to 0} \frac{\pi\cos\pi x}{2\pi\cos 2\pi x} = \frac{\pi}{2\pi} = \frac{1}{2}$$

74. $$\lim_{x\to\infty} xe^{-x^2} = \lim_{x\to\infty} \frac{x}{e^{x^2}} = \lim_{x\to\infty} \frac{1}{2xe^{x^2}} = 0$$

76. $$y = \lim_{x\to 1^+} (x - 1)^{\ln x}$$

$$\ln y = \lim_{x\to 1^+} [(\ln x)\ln(x - 1)]$$
$$= \lim_{x\to 1^+} \left[\frac{\ln(x - 1)}{\frac{1}{\ln x}}\right] = \lim_{x\to 1^+} \left[\frac{\frac{1}{x - 1}}{\left(\frac{1}{x}\right)\frac{-1}{\ln^2 x}}\right] = \lim_{x\to 1^+} \left[\frac{\frac{-\ln^2 x}{x - 1}}{x}\right] = \lim_{x\to 1^+} \left[\frac{-2\left(\frac{1}{x}\right)(\ln x)}{\frac{1}{x^2}}\right]$$
$$= \lim_{x\to 1^+} 2x(\ln x) = 0$$

Since $\ln y = 0$, $y = 1$.

78. $\lim_{x \to 1^+} \left(\frac{2}{\ln x} - \frac{2}{x-1}\right) = \lim_{x \to 1^+} \left[\frac{2x - 2 - 2\ln x}{(\ln x)(x-1)}\right]$

$= \lim_{x \to 1^+} \left[\frac{2 - (2/x)}{(x-1)(1/x) + \ln x}\right]$

$= \lim_{x \to 1^+} \frac{2x - 2}{(x-1) + x\ln x} = \lim_{x \to 1^+} \frac{2}{1 + 1 + \ln x} = 1$

80. $\int_0^1 \frac{6}{x-1}\,dx = \lim_{b \to 1^-} \Big[6\ln|x - 1|\Big]_0^b = -\infty$

Diverges

82. $\int_0^\infty \frac{e^{-1/x}}{x^2}\,dx = \lim_{\substack{a \to 0^+ \\ b \to \infty}} \Big[e^{-1/x}\Big]_a^b = 1 - 0 = 1$

84. $V = \pi\int_0^\infty (xe^{-x})^2 dx$

$= \pi\int_0^\infty x^2 e^{-2x}\,dx$

$= \lim_{b \to \infty} \left[-\frac{\pi e^{-2x}}{4}(2x^2 + 2x + 1)\right]_0^b = \frac{\pi}{4}$

86. $\int_2^\infty \left[\frac{1}{x^5} + \frac{1}{x^{10}} + \frac{1}{x^{15}}\right] dx < \int_2^\infty \frac{1}{x^5 - 1}\,dx < \int_2^\infty \left[\frac{1}{x^5} + \frac{1}{x^{10}} + \frac{2}{x^{15}}\right] dx$

$\lim_{b \to \infty} \left[-\frac{1}{4x^4} - \frac{1}{9x^9} - \frac{1}{14x^{14}}\right]_2^b < \int_2^\infty \frac{1}{x^5 - 1}\,dx < \lim_{b \to \infty} \left[-\frac{1}{4x^4} - \frac{1}{9x^9} - \frac{1}{7x^{14}}\right]_2^b$

$0.015846 < \int_2^\infty \frac{1}{x^5 - 1}\,dx < 0.015851$

Problem Solving for Chapter 7

2. (a) $\int_0^1 \ln x\,dx = \lim_{b \to 0^+} \Big[x\ln - x\Big]_b^1$

$= (-1) - \lim_{b \to 0^+} (b\ln b - b) = -1$

Note: $\lim_{b \to 0^+} b\ln b = \lim_{b \to 0^+} \frac{\ln b}{b^{-1}} = \lim_{b \to 0^+} \frac{1/b}{-1/b^2} = 0$

$\int_0^1 (\ln x)^2\,dx = \lim_{b \to 0^+} \Big[x(\ln x)^2 - 2x\ln x + 2x\Big]_b^1$

$= 2 - \lim_{b \to 0^+} (b(\ln b)^2 - 2b\ln b + 2b) = 2$

(b) Note first that $\lim_{b \to 0^+} b(\ln b)^n = 0$ (Mathematical induction).

Also, $\int (\ln x)^{n+1}\,dx = x(\ln x)^{n+1} - (n+1)\int (\ln x)^n\,dx$.

Assume $\int_0^1 (\ln x)^n\,dx = (-1)^n n!$.

Then, $\int_0^1 (\ln x)^{n+1}\,dx = \lim_{b \to 0^+} \Big[x(\ln x)^{n+1}\Big]_b^1 - (n+1)\int_0^1 (\ln x)^n\,dx$

$= 0 - (n+1)(-1)^n n! = (-1)^{n+1}(n+1)!$.

4.
$$\lim_{x\to\infty}\left(\frac{x-c}{x+c}\right)^x = \frac{1}{4}$$
$$\lim_{x\to\infty} x\ln\left(\frac{x-c}{x+c}\right) = \ln\frac{1}{4}$$
$$\lim_{x\to\infty}\frac{\ln(x-c)-\ln(x+c)}{1/x} = -\ln 4$$
$$\lim_{x\to\infty}\frac{\dfrac{1}{x-c}-\dfrac{1}{x+c}}{-\dfrac{1}{x^2}} = -\ln 4$$
$$\lim_{x\to\infty}\frac{2c}{(x-c)(x+c)}(-x^2) = -\ln 4$$
$$\lim_{x\to\infty}\frac{2cx^2}{x^2-c^2} = \ln 4$$
$$2c = \ln 4$$
$$2x = 2\ln 2$$
$$c = \ln 2$$

6. $\sin\theta = BD$, $\cos\theta = OD$

$$\text{Area }\triangle DAB = \frac{1}{2}(DA)(BD) = \frac{1}{2}(1-\cos\theta)\sin\theta$$

$$\text{Shaded area} = \frac{\theta}{2} - \frac{1}{2}(1)(BD) = \frac{\theta}{2} - \frac{1}{2}\sin\theta$$

$$R = \frac{\triangle DAB}{\text{Shaded area}} = \frac{1/2(1-\cos\theta)\sin\theta}{1/2(\theta-\sin\theta)}$$

$$\lim_{\theta\to 0^+} R = \lim_{\theta\to 0^+}\frac{(1-\cos\theta)\sin\theta}{\theta-\sin\theta} = \lim_{\theta\to 0^+}\frac{(1-\cos\theta)\cos\theta+\sin^2\theta}{1-\cos\theta}$$
$$= \lim_{\theta\to 0^+}\frac{(1-\cos\theta)(-\sin\theta)+\cos\theta\sin\theta+2\sin\theta\cos\theta}{\sin\theta}$$
$$= \lim_{\theta\to 0^+}\frac{-\sin\theta-4\cos\theta\sin\theta}{\sin\theta} = \lim_{\theta\to 0}\frac{4\cos\theta-1}{1} = 3$$

8. $u = \tan\dfrac{x}{2}$, $\cos x = \dfrac{1-u^2}{1+u^2}$, $2+\cos x = 2+\dfrac{1-u^2}{1+u^2} = \dfrac{3+u^2}{1+u^2}$

$$dx = \frac{2\,du}{1+u^2}$$

$$\int_0^{\pi/2}\frac{1}{2+\cos x}\,dx = \int_0^1\left(\frac{1+u^2}{3+u^2}\right)\left(\frac{2}{1+u^2}\right)du$$
$$= \int_0^1\frac{2}{3+u^2}\,du$$
$$= \left[2\frac{1}{\sqrt{3}}\arctan\left(\frac{u}{\sqrt{3}}\right)\right]_0^1$$
$$= \frac{2}{\sqrt{3}}\arctan\left(\frac{1}{\sqrt{3}}\right)$$
$$= \frac{2}{\sqrt{3}}\frac{\pi}{6} = \frac{\pi\sqrt{3}}{9} \approx 0.6046$$

10. Let $u = cx$, $du = c\,dx$.

$$\int_0^b e^{-c^2x^2}\,dx = \int_0^{cb} e^{-u^2}\,\frac{du}{c} = \frac{1}{c}\int_0^{cb} e^{-u^2}\,du$$

As $b \to \infty$, $cb \to \infty$. Hence, $\displaystyle\int_0^{\infty} e^{-c^2x^2}\,dx = \frac{1}{c}\int_0^{\infty} e^{-x^2}\,dx$.

$\bar{x} = 0$ by symmetry.

$$\bar{y} = \frac{M_x}{m} = \frac{2\displaystyle\int_0^{\infty}\frac{(e^{-c^2x^2})}{2}\,dx}{2\displaystyle\int_0^{\infty} e^{-c^2x^2}\,dx}$$

$$= \frac{1}{2}\,\frac{\displaystyle\int_0^{\infty} e^{-2c^2x^2}\,dx}{\displaystyle\int_0^{\infty} e^{-c^2x^2}\,dx}$$

$$= \frac{1}{2}\,\frac{\dfrac{1}{\sqrt{2}c}\displaystyle\int_0^{\infty} e^{-x^2}\,dx}{\dfrac{1}{c}\displaystyle\int_0^{\infty} e^{-x^2}\,dx}$$

$$= \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

Thus, $(\bar{x}, \bar{y}) = \left(0, \dfrac{\sqrt{2}}{4}\right)$.

12. (a) Let $y = f^{-1}(x)$, $f(y) = x$, $dx = f'(y)\,dy$.

$$\int f^{-1}(x)\,dx = \int y f'(y)\,dy$$

$$= yf(y) - \int f(y)\,dy \qquad \left[\begin{array}{l} u = y,\ du = dy \\ dv = f'(y)\,dy,\ v = f(y) \end{array}\right]$$

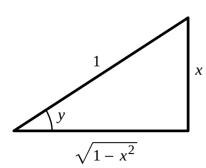
$$= xf^{-1}(x) - \int f(y)\,dy$$

(b) $f^{-1}(x) = \arcsin x = y$, $f(x) = \sin x$

$$\int \arcsin x\,dx = x\arcsin x - \int \sin y\,dy$$

$$= x\arcsin x + \cos y + C$$

$$= x\arcsin x + \sqrt{1 - x^2} + C$$



(c) $f(x) = e^x$, $f^{-1}(x) = \ln x = y$ $\qquad x = 1 \Leftrightarrow y = 0$; $x = e \Leftrightarrow y = 1$

$$\int_1^e \ln x\,dx = \Big[x\ln x\Big]_1^e - \int_0^1 e^y\,dy$$

$$= e - \Big[e^y\Big]_0^1$$

$$= e - (e - 1) = 1$$

14. (a) Let $x = \frac{\pi}{2} - u$, $dx = du$.

$$I = \int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x}\,dx = \int_{\pi/2}^{0} \frac{\sin\left(\frac{\pi}{2} - u\right)}{\cos\left(\frac{\pi}{2} - u\right) + \sin\left(\frac{\pi}{2} - u\right)}(-du)$$

$$= \int_0^{\pi/2} \frac{\cos u}{\sin u + \cos u}\,du$$

Hence,

$$2I = \int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x}\,dx + \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x}\,dx$$

$$= \int_0^{\pi/2} 1\,dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}.$$

(b) $$I = \int_{\pi/2}^{0} \frac{\sin^n\left(\frac{\pi}{2} - u\right)}{\cos^n\left(\frac{\pi}{2} - u\right) + \sin^n\left(\frac{\pi}{2} - u\right)}(-du)$$

$$= \int_0^{\pi/2} \frac{\cos^n u}{\sin^n u + \cos^n u}\,du$$

Thus, $2I = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$.

16. $$\frac{N(x)}{D(x)} = \frac{P_1}{x - c_1} + \frac{P_2}{x - c_2} + \cdots + \frac{P_n}{x - c_n}$$

$$N(x) = P_1(x - c_2)(x - c_3)\ldots(x - c_n) + P_2(x - c_1)(x - c_3)\ldots(x - c_n) + \cdots + P_n(x - c_1)(x - c_2)\ldots(x - c_{n-1})$$

Let $x = c_1$: $N(c_1) = P_1(c_1 - c_2)(c_1 - c_3)\ldots(c_1 - c_n)$

$$P_1 = \frac{N(c_1)}{(c_1 - c_2)(c_1 - c_3)\ldots(c_1 - c_n)}$$

Let $x = c_2$: $N(c_2) = P_2(c_2 - c_1)(c_2 - c_3)\ldots(c_2 - c_n)$

$$P_2 = \frac{N(c_2)}{(c_2 - c_1)(c_2 - c_3)\ldots(c_2 - c_n)}$$

$\vdots$ $\vdots$

Let $x = c_n$: $N(c_n) = P_n(c_n - c_1)(c_n - c_2)\ldots(c_n - c_{n-1})$

$$P_n = \frac{N(c_n)}{(c_n - c_1)(c_n - c_2)\ldots(c_n - c_{n-1})}$$

If $D(x) = (x - c_1)(x - c_2)(x - c_3)\ldots(x - c_n)$, then by the Product Rule

$$D'(x) = (x - c_2)(x - c_3)\ldots(x - c_n) + (x - c_1)(x - c_3)\ldots(x - c_n) + \cdots + (x - c_1)(x - c_2)(x - c_3)\ldots(x - c_{n-1})$$

and

$$D'(c_1) = (c_1 - c_2)(c_1 - c_3)\ldots(c_1 - c_n)$$

$$D'(c_2) = (c_2 - c_1)(c_2 - c_3)\ldots(c_2 - c_n)$$

$$\vdots$$

$$D'(c_n) = (c_n - c_1)(c_n - c_2)\ldots(c_n - c_{n-1}).$$

Thus, $P_k = N(c_k)/D'(c_k)$ for $k = 1, 2, \ldots, n$.

18. $$s(t) = \int \left[-32t + 12{,}000 \ln \frac{50{,}000}{50{,}000 - 400t} \right] dt$$

$$= -16t^2 + 12{,}000 \int [\ln 50{,}000 - \ln(50{,}000 - 400t)]\, dt$$

$$= 16t^2 + 12{,}000t \ln 50{,}000 - 12{,}000\left[t \ln(50{,}000 - 400t) - \int \frac{-400t}{50{,}000 - 400t}\, dt \right]$$

$$= -16t^2 + 12{,}000t \ln \frac{50{,}000}{50{,}000 - 400t} + 12{,}000t \int \left[1 - \frac{50{,}000}{50{,}000 - 400t} \right] dt$$

$$= -16t^2 + 12{,}000t \ln \frac{50{,}000}{50{,}000 - 400t} + 12{,}000t + 1{,}500{,}000 \ln(50{,}000 - 400t) + C$$

$$s(0) = 1{,}500{,}000 \ln 50{,}000 + C = 0$$

$$C = -1{,}500{,}000 \ln 50{,}000$$

$$s(t) = -16t^2 + 12{,}000t \left[1 + \ln \frac{50{,}000}{50{,}000 - 400t} \right] + 1{,}500{,}000 \ln \frac{50{,}000 - 400t}{50{,}000}$$

When $t = 100$, $s(100) \approx 557{,}168.626$ feet

20. Let $u = (x - a)(x - b)$, $du = [(x - a) + (x - b)]\, dx$, $dv = f''(x)\, dx$, $v = f'(x)$.

$$\int_a^b (x - a)(x - b)\, dx = \Big[(x - a)(x - b) f'(x) \Big]_a^b - \int_a^b [(x - a) + (x - b)] f'(x)\, dx$$

$$= -\int_a^b (2x - a - b) f'(x)\, dx \qquad \begin{pmatrix} u = 2x - a - b \\ dv = f'(x)\, dx \end{pmatrix}$$

$$= \Big[-(2x - a - b) f(x) \Big]_a^b + \int_a^b 2f(x)\, dx$$

$$= 2\int_a^b f(x)\, dx$$